NET DECEMBER-2019

PART A

Q1. A two-digit number is such that if the digit 4 is placed to its right, its value would increase by 490. Find the original number.

(a) 48 (b) 54 (c) 64 (d) 56

Q2. Given that $K! = 1\times2\times3\times...\times K$, which is the largest among the following numbers?

(a) $(2!)^{1/2}$ (b) $(3!)^{1/3}$ (c) $(4!)^{1/4}$ (d) $\frac{(3!)}{2}$

Q3. Of three children, Uma plays all three of cricket, football and hockey. Iqbal plays cricket but not football and Tarun plays hockey but neither football nor cricket. The number of games played by at least two of the children is

(a) One (b) Two (c) Three (d) zero

Q4. A multiple choice exam has 4 questions, each with 4 answer choices. Every question has only one correct answer. The probability of getting all answers correct by independent random guesses for each one is

(a) 14 (b) $(1/4)^4$ (c) $(3/4)$ (d) $(3/4)^4$

Q5. The result of a survey to find the most preferred leader among A, B, C is shown in the table

Votes	A	B	C
1^{st} preference	13	54	33
2^{nd} preference	24	37	39
3^{rd} preference	63	9	28

First, second and third preferences are given weights $3,2,1$, respectively. Statistically, which of the following can be said to represent the preferences of the voters?

(a) A and C are within 10% of each other

(b) B is the most preferred

(c) B and C are within 10% of each other

(d) C is the most preferred

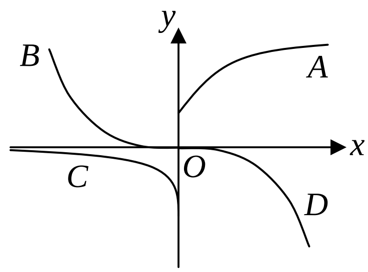
Q6. Which is the curve in the figure whose points satisfy the equation $y = \text{constant} \times e^{x}$?

(a) A

(b) B

(c) C

(d) D



Q7. An ice cube of volume $10\ cm^3$ is floating over a glass of water of $10\ cm^2$ cross-section area and $10\ cm$ height. The level of the water is exactly at the brim of the glass. Given that the density of ice is 10% less than that of water, what will be the situation when ice melts completely?

(a) the level falls by 10% of the side of the cube.

(b) The level falls by 10% of the original height of the water column

(c) The level increases by 10% of the side of the cube and water spills out

(d) There is no change in the level of the water.

Q8. In a college admission where applicants have to choose only one subject, $1/4^{th}$ of the applicants opted for Biology. $1/6^{th}$ for chemistry, $1/8^{th}$ for Physics and $1/12^{th}$ for Maths. 18 applicants did not opt for any of the above four subjects. How many applicants were there?

(a) 22 (b) 24 (c) 36 (d) 48

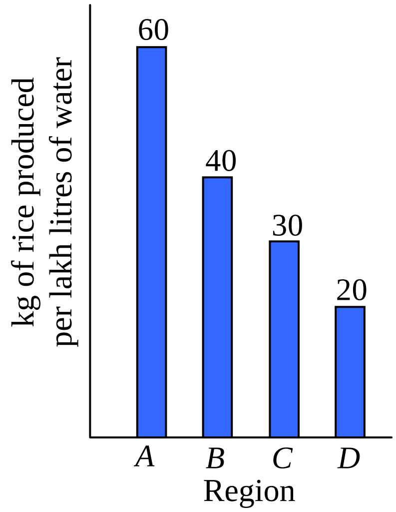
Q9. Based on the bar chart shown here, which of the following inferences is correct?

(a) Region A uses maximum water per kg of rice

(b) Average water consumption of the four regions is 37.5 lakh liters

(c) Region D uses thrice the amount of water used by region A per kg of rice.

(d) Region B uses 20 lakh litres of less water than region A



Q10. In a race five drivers were in the following situation. M was following V, R was just ahead of T and K was the only one between T and V. Who was in the second place at that instant?

(a) V (b) R (c) T (d) K

Q11. A bag contains 8 red balls, 17 green balls. What is the minimum number of balls that needs to be taken out from the bag to ensure getting at least one ball of each colour?

(a) 19 (b) 18 (c) 28 (d) 27

Q12. In a very old, stable forest, a particular species of plants grows to a maximum height of $3m$. In a large survey, it is found that 30% of the plants have heights less than $1\,m$ and 50% have heights more than $2\,m$. From these observations we can say that the height of the plants increases

(a) at the slowest rate when they are less than $1m$ tall

(b) at the fastest rate when they are between $1m$ and $2\,m$ tall

(c) at the fastest rate when they are more than $2\,m$ tall

(d) at the same rate at all stages

Q13. What day of the week will it be 61 days from a Friday?

(a) Saturday (b) Sunday (c) Friday (d) Wednesday

Q14. Which of the following 7-digit numbers CANNOT be perfect squares?

$$A = 45xyz26,\ B = 2xyz175,\ C = xyz3310$$

(a) Only A (b) Only B (c) Only C (d) All three

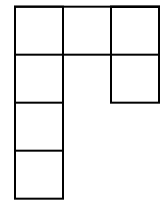
Q15. A cyclist covers a certain distance at a constant speed. If a jogger covers half the distance in double the time as the cyclist, the ratio of the speed of the jogger to that of the cyclist is

(a) 1:4 (b) 4:1 (c) 1:2 (d) 2:1

Q16. What is the ratio of the surface area of a cube with side $1cm$ to the total surface area of the cubes formed by breaking the original cube into identical cubes of side $1mm$?

(a) $\frac{1}{6}$ (b) $\frac{1}{10}$ (c) $\frac{1}{100}$ (d) $\frac{1}{36}$

Q17. How many non-square rectangles are there in the following figure, consisting of 7 squares?



(a) 8 (b) 9 (c) 10 (d) 11

Q18. The mean of a set of 10 numbers is M. By combining with it a second set of M numbers, the mean of the combined set becomes 10. What is the sum of the second set of numbers?

(a) $10M-1$ (b) $10M+1$ (c) 20 (d) 100

Q19. Karan's house is $20\,m$ to the east of Rahul's house. Mehul's house is $25\,m$ to the North-East of Rahul's house. With respect to Mehul's house in which direction is Karan's house?

(a) East (b) South (c) North-East (d) West

Q20. A four-wheeled cart is going around a circular track. Which of the following statements is correct, if the four wheels are free to rotate independent of each other and the cart negotiates the track stably?

(a) All wheels rotate at the same speed

(b) The four wheels have different speeds each

(c) The wheels closer to the inside of the track move slower than the outer-side wheels

(d) The wheels closer to the inside of the track move faster than the outer-side wheels

PART B

Q21. The angular frequency of oscillation of a quantum harmonic oscillator in two dimensions is ω. If it is in contact with an external heat bath at temperature T, its partition function is (in the following $\beta = \frac{1}{k_B T}$)

(a) $\frac{e^{2\beta\hbar\omega}}{\left(e^{2\beta\hbar\omega}-1\right)^2}$ (b) $\frac{e^{\beta\hbar\omega}}{\left(e^{\beta\hbar\omega}-1\right)^2}$ (c) $\frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ (d) $\frac{e^{2\beta\hbar\omega}}{e^{2\beta\hbar\omega}-1}$

Q22. A student measures the displacement x from the equilibrium of a stretched spring and reports it be $100\,\mu m$ with a 1% error. The spring constant k is known to be $10\,N/m$ with 0.5% error. The percentage error in the estimate of the potential energy $V = \frac{1}{2}kx^2$ is

(a) 0.8% (b) 2.5% (c) 1.5% (d) 3.0%

Q23. The Hamiltonian of two interacting particles one with spin 1 and the other with spin $\frac{1}{2}$ is given by $H = A\vec{S}_1 \cdot \vec{S}_2 + B\left(S_{1x} + S_{2x}\right)$, where $\vec{S}_1$ and $\vec{S}_2$ denote the spin operators of the first and second particles, respectively and A and B are positive constants. The largest eigenvalue of this Hamiltonian is

(a) $\frac{1}{2}\left(A\hbar^2 + 3B\hbar\right)$ (b) $3A\hbar^2 + B\hbar$ (c) $\frac{1}{2}\left(3A\hbar^2 + B\hbar\right)$ (d) $A\hbar^2 + 3B\hbar$

Q24. Consider the set of polynomials $\left\{x(t) = a_0 + a_1 t + \ldots + a_{n-1}t^{n-1}\right\}$ in t of degree less than n, such that $x(0) = 0$ and $x(1) = 1$. This set

(a) constitutes a vector space of dimension n

(b) constitutes a vector space of dimension $n-1$

(c) constitutes a vector space of dimension $n-2$

(d) does not constitute a vector space

Q25. Consider black body radiation in thermal equilibrium contained in a two-dimensional box. The dependence of the energy density on the temperature T is

(a) T^3 (b) T (c) T^2 (d) T^4

Q26. The energy eigenvalues of a particle of mass m, confined to a rigid one-dimensional box of width L, are $E_n\left(n=1,2,...\right)$. If the walls of the box are moved very slowly toward each other, the rate of change of time-dependent energy $\frac{dE_2}{dt}$ of the first excited state is

(a) $\frac{E_2}{L}\frac{dL}{dt}$ (b) $\frac{2E_2}{L}\frac{dL}{dt}$ (c) $-\frac{2E_2}{L}\frac{dL}{dt}$ (d) $-\frac{E_1}{L}\frac{dL}{dt}$

Q27. A ball, initially at rest, is dropped from a height h above the floor bounces again and again vertically. If the coefficient of restitution between the ball and the floor is 0.5, the total distance traveled by the ball before it comes to rest is

(a) $\frac{8h}{3}$ (b) $\frac{5h}{3}$ (c) $3h$ (d) $2h$

Q28. Two spin $\frac{1}{2}$ fermions of mass m are confined to move in a one-dimensional infinite potential well of width L. If the particles are known to be in a spin triplet state, the ground state energy of the system (in units of $\frac{\hbar^2\pi^2}{2mL^2}$) is

(a) 8 (b) 2 (c) 3 (d) 5

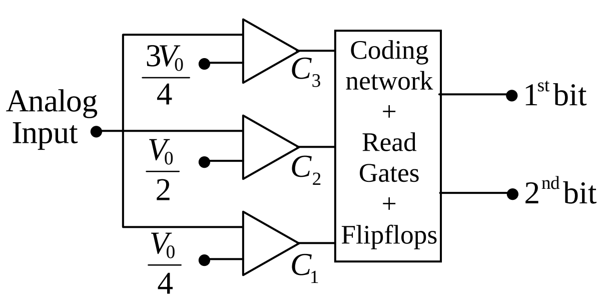
Q29. The figure below shows a 2-bit simultaneous analog-to-digital (A/D) converter operating in the voltage range 0 to V_0. The output of the comparators are C_1, C_2 and C_3 with the reference inputs $V_0/4, V_0/2$ and $3V_0/4$, respectively. The logic expression for the output corresponding to the less significant bit is

(a) $C_1C_2C_3$

(b) $C_2\bar{C}_3+\bar{C}_1$

(c) $C_1\bar{C}_2+C_3$

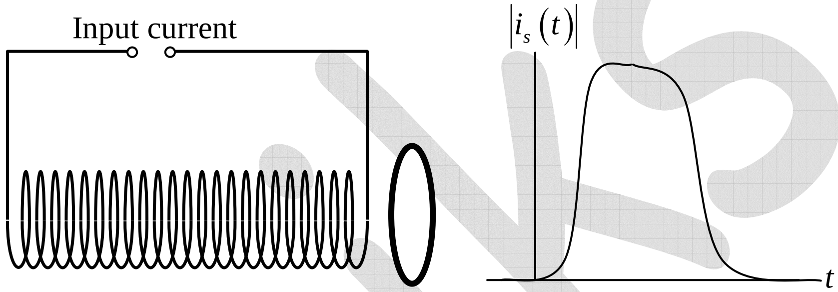
(d) $C_2\bar{C}_3+C_2$



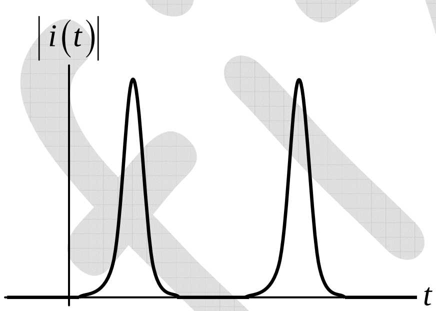
Q30. The yz - plane at $x = 0$ carries a uniform surface charge density σ. A unit point charge is moved from a point $(\delta, 0, 0)$ on one side of the plane to a point $(-\delta, 0, 0)$ on the other side. If δ is an infinitesimally small positive number, the work done in moving the charge is

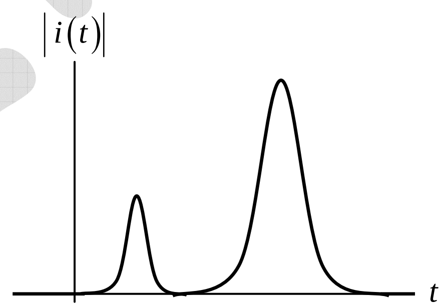
(a) 0 (b) $\frac{\sigma}{\varepsilon_0}\delta$ (c) $-\frac{\sigma}{\varepsilon_0}\delta$ (d) $\frac{2\sigma}{\varepsilon_0}\delta$

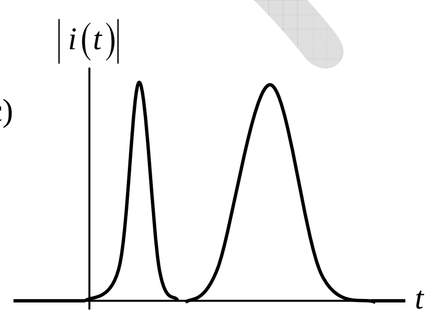
Q31. A circular conducting wire loop is placed close to a solenoid as shown in the figure below. Also shown is the current through the solenoid as a function of time.



The magnitude $|i(t)|$ of the induced current in the wire loop, as a function of time t, is best represented as

(a)


(b)


(c)


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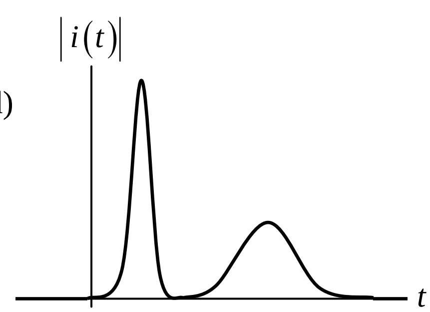


Q32. A mole of gas at initial temperature T_i comes into contact with a heat reservoir at temperature T_f and the system is allowed to reach equilibrium at constant volume. If the specific heat of the gas is $C_V = \alpha T$, where α is a constant, the total change in entropy is

(a) zero

(b) $\alpha\left(T_f - T_i\right) + \frac{\alpha}{2T_f}\left(T_f - T_i\right)^2$

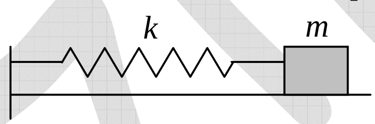
(c) $\alpha\left(T_f - T_i\right)$

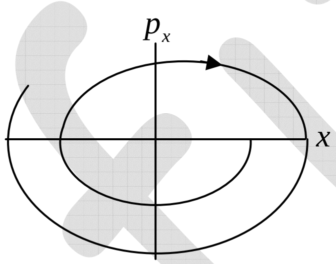
(d) $\alpha\left(T_f - T_i\right) + \frac{\alpha}{2T_f}\left(T_f^2 - T_i^2\right)$

Q33. An ideal Carnot engine extracts $100\ J$ from a heat source and dumps $40\ J$ to a heat sink at $300\ K$. The temperature of the heat source is

(a) $600\ K$ (b) $700\ K$ (c) $750\ K$ (d) $650\ K$

Q34. A block of mass m, attached to a spring, oscillates horizontally on a surface. The coefficient of friction between the block and the surface is μ. Which of the following trajectories best describes the motion of the block in the phase space (xp_x -plane)?



(a)


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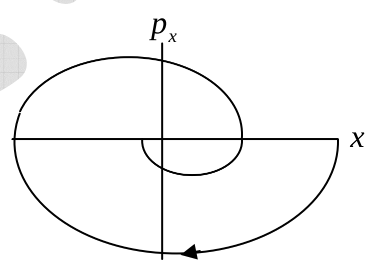

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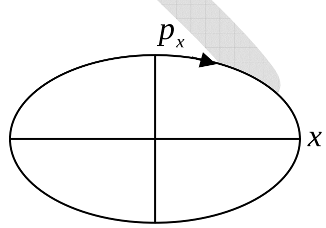

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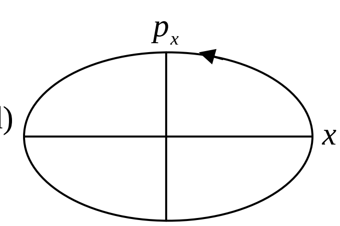


Q35. Let C be the circle of radius $\frac{\pi}{4}$ centered at $z=\frac{1}{4}$ in the complex z-plane that is traversed counter-clockwise. The value of the contour integral $\oint_C \frac{z^2}{\sin^2 4z} dz$ is

(a) 0 (b) $\frac{i\pi^2}{4}$ (c) $\frac{i\pi^2}{16}$ (d) $\frac{i\pi}{4}$

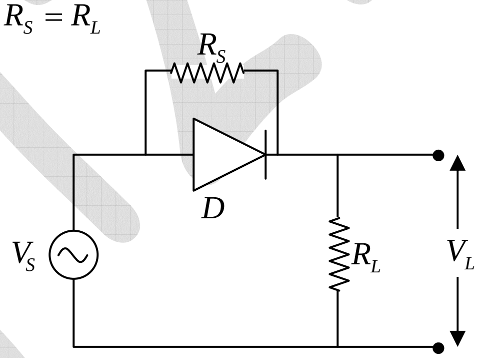
Q36. If the rank of an $n \times n$ matrix A is m, where m and n are positive integers with $1 \le m \le n$, then the rank of the matrix A^2 is

(a) m (b) $m-1$ (c) $2m$ (d) $m-2$

Q37. A particle of mass m is confined to a box of unit length in one dimension. It is described by the wavefunction $\psi(x)=\sqrt{\frac{8}{5}}\sin \pi x(1+\cos \pi x)$ for $0 \le x \le 1$ and zero outside this interval. The expectation value of energy in this state is

(a) $\frac{4\pi^2}{3m}\hbar^2$ (b) $\frac{4\pi^2}{5m}\hbar^2$ (c) $\frac{2\pi^2}{5m}\hbar^2$ (d) $\frac{8\pi^2}{5m}\hbar^2$

Q38. In the circuit below, D is an ideal diode, the source voltage $V_S = V_0 \sin \omega t$ is a unit amplitude sine wave and $R_S = R_L$



The average output voltage, V_L, across the load resistor R_L is

(a) $\frac{1}{2\pi}V_0$ (b) $\frac{3}{2\pi}V_0$ (c) $3V_0$ (d) V_0

Q39. The normalized wavefunction of a particle in three dimensions is given by

$$\psi(x,y,z) = N\,z\exp\left[-a\left(x^2+y^2+z^2\right)\right]$$

where a is a positive constant and N is a normalization constant. If L is the angular momentum operator, the eigenvalues of L^2 and L_z, respectively, are

(a) $2\hbar^2$ and $\hbar$ (b) $\hbar^2$ and 0 (c) $2\hbar^2$ and 0 (d) $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$

Q40. The electric field of an electromagnetic wave is $\vec{E} = \hat{i}\sqrt{2}\sin(kz-\omega t)\,Vm^{-1}$. The average flow of energy per unit area per unit time, due to this wave, is

(a) $27\times10^{4}\,W/m^2$ (b) $27\times10^{-4}\,W/m^2$

(c) $27\times10^{-2}\,W/m^2$ (d) $27\times10^{2}\,W/m^2$

Q41. The energies available to a three state system are $0, E$ and $2E$, where $E>0$. Which of the following graphs best represents the temperature dependence of the specific heat?

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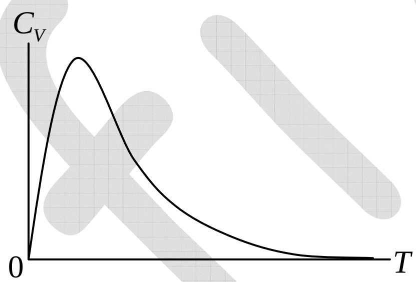

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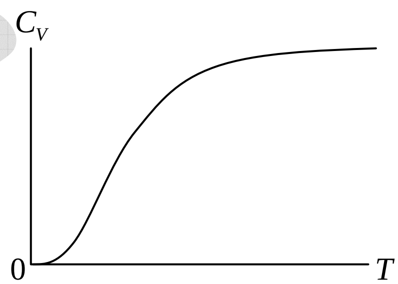

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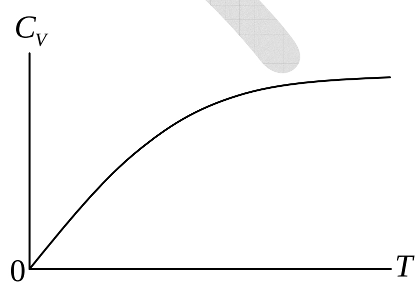

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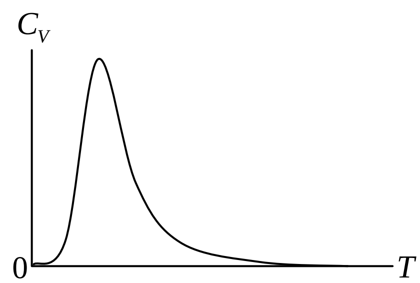

Q42. The values of a and b for which the force $F = \left(axy + z^3\right)\hat{i} + x^2\hat{j} + bxz^2\hat{k}$ is conservative are

(a) $a = 2, b = 3$ (b) $a = 1, b = 3$ (c) $a = 2, b = 6$ (d) $a = 3, b = 2$

Q43. A positively charged particle is placed at the origin (with zero initial velocity) in the presence of a constant electric and a constant magnetic field along the positive z and x-directions, respectively. At large times, the overall motion of the particle is adrift along the

(a) positive y - direction (b) negative z - direction

(c) positive z - direction (d) negative y - direction

Q44. A box contains 5 white and 4 black balls. Two balls are picked together at random from the box. What is the probability that these two balls are of different colours?

(a) $\frac{1}{2}$ (b) $\frac{5}{18}$ (c) $\frac{1}{3}$ (d) $\frac{5}{9}$

Q45. Which of the following terms, when added to the Lagrangian $L\left(x, y, \dot{x}, \dot{y}\right)$ of a system with two degrees of freedom will not change the equations of motion?

(a) $x\ddot{x} - y\ddot{y}$ (b) $x\ddot{y} - y\ddot{x}$ (c) $x\dot{y} - y\dot{x}$ (d) $y\dot{x}^2 + x\dot{y}^2$

(check question)

PART C

Q46. The outermost shell of an atom of an element is $3d^3$. The spectral symbol for the ground state is

(a) $^4F_{3/2}$ (b) $^4F_{9/2}$ (c) $^4D_{7/2}$ (d) $^4D_{1/2}$

Q47. In a spectrum resulting from Raman scattering, let I_R denote the intensity of Rayleigh scattering and I_S and I_{AS} denote the most intense Stokes line and the most intense anti-Stokes line, respectively. The correct order of these intensities is

(a) $I_S > I_R > I_{AS}$ (b) $I_R > I_S > I_{AS}$ (c) $I_{AS} > I_R > I_S$ (d) $I_R > I_{AS} > I_S$

Q48. A particle hops randomly from a site to its nearest neighbour in each step on a square lattice of unit lattice constant. The probability of hopping to the positive x-direction is 0.3, to the negative x-direction is 0.2, to the positive y-direction is 0.2 and to the negative y-direction is 0.3. If a particle starts from the origin, its mean position after N steps is

(a) $\frac{1}{10}N\left(-\hat{i}+\hat{j}\right)$ (b) $\frac{1}{10}N\left(\hat{i}-\hat{j}\right)$ (c) $N\left(0.3\hat{i}-0.2\hat{j}\right)$ (d) $N\left(0.2\hat{i}-0.3\hat{j}\right)$

Q49. Let $\hat{x}$ and $\hat{p}$ denote position and momentum operators obeying the commutation relation $\left[\hat{x},\hat{p}\right]=i\hbar$. If $|x\rangle$ denotes an eigenstate of $\hat{x}$ corresponding to the eigenvalue x, then $e^{ia\hat{p}/\hbar}|x\rangle$ is

(a) an eigenstate of $\hat{x}$ corresponding to the eigenvalue x

(b) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x+a)$

(c) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x-a)$

(d) not an eigenstate of $\hat{x}$

Q50. The strong nuclear force between a neutron and a proton in a zero orbital angular momentum state is denoted by $F_{np}(r)$, where r is the separation between them. Similarly, $F_{nn}(r)$ and $F_{pp}(r)$ denote the forces between a pair of neutrons and protons, respectively, in zero orbital momentum state. Which of the following is true on average if the inter-nucleon distance is $0.2\ fm < r < 2\ fm$?

(a) F_{np} is attractive for triplet spin state, and F_{nn}, F_{pp} are always repulsive

(b) F_{nn} and F_{np} are always attractive and F_{pp} is repulsive in the triplet spin state

(c) F_{pp} and F_{np} are always attractive and F_{nn} is always repulsive

(d) All three forces are always attractive

Q51. The Hall coefficient for a semiconductor having both types of carriers is given as

$$R_H = \frac{p\mu_p^2 - n\mu_n^2}{|e|\left(p\mu_p + n\mu_n\right)^2}$$

where p and n are the carrier densities of the holes and electrons, μ_p and μ_n are their respective mobilities. For a p - type semiconductor in which the mobility of holes is less than that of electrons, which of the following graphs best describes the variation of the Hall coefficient with temperature?

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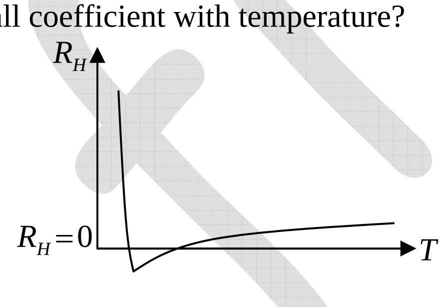

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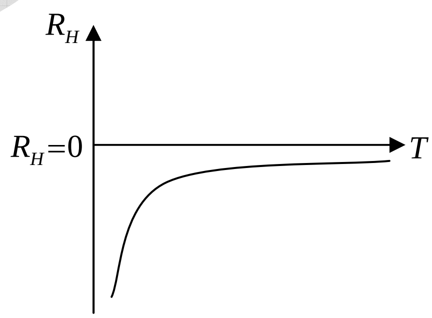

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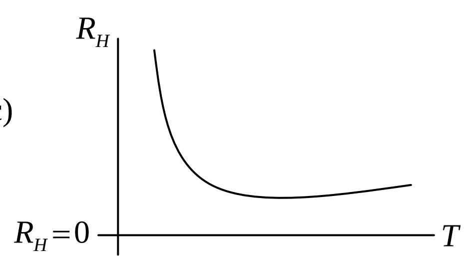

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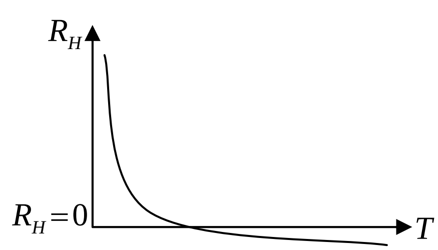


Q52. The generator of the infinitesimal canonical transformation $q \to q' = (1+\in)q$ and $p \to p' = (1-\in)p$ is

(a) $q+p$ (b) qp (c) $\frac{1}{2}(q^2-p^2)$ (d) $\frac{1}{2}(q^2+p^2)$

Q53. Assume that the noise spectral density, at any given frequency, in a current amplifier is independent of frequency. The bandwidth of measurement is changed from $1\,Hz$ to $10\,Hz$. The ratio A/B of the RMS noise current before (A) and after (B) the bandwidth modification is

(a) $1/10$ (b) $1/\sqrt{10}$ (c) $\sqrt{10}$ (d) 10

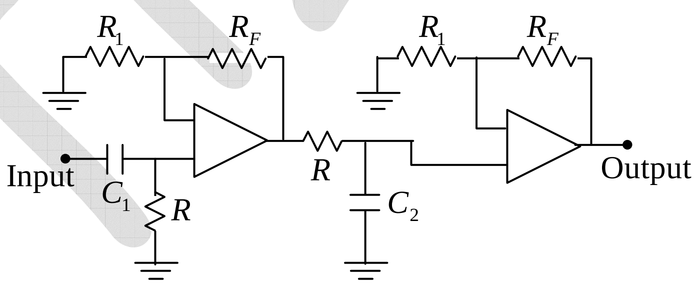
(a) $\frac{4}{3}$ and $\frac{1}{3}$ (b) $\frac{4}{3}$ and $\frac{2}{3}$ (c) 2 and $\frac{2}{3}$ (d) 2 and 1

Q55. For a crystal, let ϕ denote the energy required to create a pair of vacancy and interstitial defects. If n pairs of such defects are formed, and $n \ll N, N'$, where N and N' are respectively, the total number of lattice and interstitial sites, then n is approximately

(a) $\sqrt{NN'}e^{-\phi/(2k_BT)}$ (b) $\sqrt{NN'}e^{-\phi/(k_BT)}$

(c) $\frac{1}{2}(N+N')e^{-\phi/(2k_BT)}$ (d) $\frac{1}{2}(N+N')e^{-\phi/(k_BT)}$

Q56. In the circuit diagram of a band pass filter shown below, $R = 10\,k\Omega$.



In order to get a lower cut-off frequency of $150\,Hz$ and an upper cut-off frequency of $10\,kHz$, the appropriate values of C_1 and C_2 respectively are

(a) $0.1\,\mu F$ and $1.5\,nF$ (b) $0.3\,\mu F$ and $5.0\,nF$

(c) $1.5\,nF$ and $0.1\,\mu F$ (d) $5.0\,nF$ and $0.3\,\mu F$

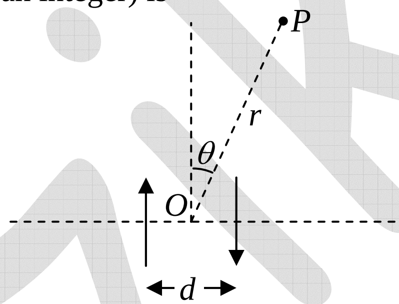
Q57. The Bethe-Weizsacker formula for the binding energy (in MeV) of a nucleus of atomic number Z and mass number A is

$$15.8A - 18.3A^{2/3} - 0.714\frac{Z(Z-1)}{A^{1/3}} - 23.2\frac{(A-2Z)^2}{A}$$

The ratio Z/A for the most stable isobar of a $A = 64$ nucleus, is nearest to

(a) 0.30 (b) 0.35 (c) 0.45 (d) 0.50

Q58. The phase difference between two small oscillating electric dipoles, separated by a distance d, is π. If the wavelength of the radiation is λ, the condition for constructive interference between the two dipolar radiations at a point P when $r \gg d$ (symbols are as shown in the figure and n is an integer) is



(a) $d\sin\theta = \left(n+\frac{1}{2}\right)\lambda$ (b) $d\sin\theta = n\lambda$

(c) $d\cos\theta = n\lambda$ (d) $d\cos\theta = \left(n+\frac{1}{2}\right)\lambda$

Q59. The Hamiltonian of two particles, each of mass m, is

$H(q_1, p_1; q_2, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + k\left(q_1^2 + q_2^2 + \frac{1}{4}q_1 q_2\right)$, where $k > 0$ is a constant. The value of the partition function

$$Z(\beta) = \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dp_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} dp_2 e^{-\beta H(q_1, p_1; q_2, p_2)}$$ is

(a) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{16}{15}}$ (b) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{15}{16}}$ (c) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{63}{64}}$ (d) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{64}{63}}$

Q60. In the AC Josephson effect, a supercurrent flows across two superconductors separated by a thin insulating layer and kept at an electric potential difference ΔV. The angular frequency of the resultant supercurrent is given by

(a) $\frac{2e\Delta V}{\hbar}$ (b) $\frac{e\Delta V}{\hbar}$ (c) $\frac{e\Delta V}{\pi\hbar}$ (d) $\frac{e\Delta V}{2\pi\hbar}$

Q61. A negative muon, which has a mass nearly 200 times that of an electron, replaces an electron in a Li atom. The lowest ionization energy for the muonic Li atom is approximately

(a) the same as that of He

(b) the same as that of normal Li

(c) 200 times larger than that of normal Li

(d) the same as that of normal Be

Q62. The wavefunction of a particle of mass m, constrained to move on a circle of unit radius centered at the origin in the xy- plane, is described by $\psi(\phi) = A\cos^2\phi$, where ϕ is the azimuthal angle. All the possible outcomes of measurements of the z- component of the angular momentum L_z in this state, in units of $\hbar$ are

(a) ± 1 and 0 (b) ± 1 (c) ± 2 (d) ± 2 and 0

Q63. An alternating current $I(t) = I_0 \cos(\omega t)$ flows through a circular wire loop of radius R, lying in the xy-plane, and centered at the origin. The electric field $\vec{E}(\vec{r},t)$ and the magnetic field $\vec{B}(\vec{r},t)$ are measured at a point $\vec{r}$ such that $r \gg \frac{c}{\omega} \gg R$, where $\vec{r} = |\vec{r}|$.

Which one of the following statements is correct?

(a) The time-averaged $\left|\vec{E}(\vec{r},t)\right| \propto \frac{1}{r^2}$

(b) The time-averaged $\left|\vec{E}(\vec{r},t)\right| \propto \omega^2$

(c) The time-averaged $\left|\vec{B}(\vec{r},t)\right|$ as a function of the polar angle θ has a minimum at $\theta = \frac{\pi}{2}$

(d) $\vec{B}(\vec{r},t)$ is along the azimuthal direction

Q64. The positive zero of the polynomial $f(x) = x^2 - 4$ is determined using Newton-Raphson method, using initial guess $x = 1$. Let the estimate, after two iterations, be $x^{(2)}$. The percentage error $\left|\frac{x^{(2)} - 2}{2}\right| \times 100\%$ is

(a) 7.5% (b) 5.0% (c) 1.0% (d) 2.5%

Q65. Which of the following decay processes is allowed?

(a) $K^0 \to \mu^+ + \mu^-$ (b) $\mu^- \to e^- + \gamma$

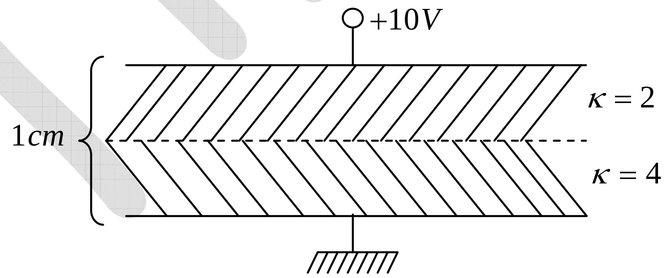
(c) $n \to p + \pi^-$ (d) $n \to \pi^+ + \pi^-$

Q66. A metallic wave guide of square cross-section of side L is excited by an electromagnetic wave of wave-number k. The group velocity of the TE_{11} mode is

(a) $\frac{ckL}{\sqrt{k^2L^2 + \pi^2}}$ (b) $\frac{c}{kL}\sqrt{k^2L^2 - 2\pi^2}$

(c) $\frac{c}{kL}\sqrt{k^2L^2 - \pi^2}$ (d) $\frac{ckL}{\sqrt{k^2L^2 + 2\pi^2}}$

Q67. A parallel plate capacitor with 1 cm separation between the plates has two layers of dielectric with dielectric constants $\kappa = 2$ and $\kappa = 4$, as shown in the figure below. If a potential difference of $10V$ is applied between the plates, the magnitude of the bound surface charge density (in units of C/m^2) at the junction of the dielectrics is



(a) $250\varepsilon_0$ (b) $2000\varepsilon_0/3$ (c) $2000\varepsilon_0$ (d) $200\varepsilon_0/3$

Q68. The Hamiltonian of a system with two degrees of freedom is $H = q_1p_1 - q_2p_2 + aq_1^2$, where $a > 0$ is a constant. The function $q_1q_2 + \lambda p_1p_2$ is a constant of motion only if λ is

(a) 0 (b) 1 (c) $-a$ (d) a

Q69. The function $f(t)$ is a periodic function of period 2π. In the range $(-\pi,\pi)$, it equals e^{-t}. If $f(t)=\sum_{-\infty}^{\infty} c_n e^{int}$ denotes its Fourier series expansion, the sum $\sum_{-\infty}^{\infty}|c_n|^2$ is

(a) 1 (b) $\frac{1}{2\pi}$ (c) $\frac{1}{2\pi}\cosh(2\pi)$ (d) $\frac{1}{2\pi}\sinh(2\pi)$

Q70. The fixed points of the time evolution of a one-variable dynamical system described by $y_{t+1}=1-2y_t^2$ are 0.5 and -1. The fixed points 0.5 and -1 are

(a) both stable (b) both unstable

(c) unstable and stable, respectively (d) stable and unstable, respectively

Q71. Following a nuclear explosion, a shock wave propagates radially outwards. Let E be the energy released in the explosion and ρ be the mass density of the ambient air. Ignoring the temperature of the ambient air, using dimensional analysis, the functional dependence of the radius R of the shock front on E,ρ and the time t is

(a) $\left(\frac{Et^2}{\rho}\right)^{1/5}$ (b) $\left(\frac{\rho}{Et^2}\right)^{1/5}$ (c) $\frac{Et^2}{\rho}$ (d) $E\rho t^2$

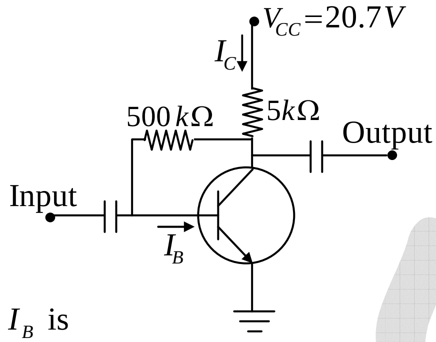
Q72. The pressure p of a gas depends on the number density ρ of particles and the temperature T as $p=k_BT\rho-B_2\rho^2+B_3\rho^3$ where B_2 and B_3 are positive constants. Let T_c, ρ_c and p_c denote the critical temperature, critical number density and critical pressure, respectively. The ratio $\rho_c k_B T_c / p_c$ is equal to

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{8}{3}$ (d) 4

Q73. The mean kinetic energy per atom in a sodium vapour lamp is $0.33eV$. Given that the mass of sodium is approximately $22.5\times10^9 eV$, the ratio of the Doppler width of an optical line to its central frequency is

(a) 7×10^{-7} (b) 6×10^{-6} (c) 5×10^{-5} (d) 4×10^{-4}

Q74. In a collector feedback circuit shown in the figure below, the base emitter voltage $V_{BE} = 0.7V$ and current gain $\beta = \frac{I_C}{I_B} = 100$ for the transistor



The value of the base current I_B is

(a) 20 μA (b) 40 μA (c) 10 μA (d) 100 μA

Q75. For T much less than the Debye temperature of copper, the temperature dependence of the specific heat at constant volume of copper, is given by (in the following a and b are positive constants)

(a) aT^3 (b) $aT + bT^3$ (c) $aT^2 + bT^3$ (d) $\exp\left(-\frac{a}{k_B T}\right)$
