

NET JUNE 2019

PART- A

- Q1.** In a bacterial cell, a protein is synthesized at random location in the cytoplasm. The protein has to reach one pole of the cell for its appropriate function. The protein reaches the pole by
- (a) Chemical attraction (b) Random movement
(c) Enzymatic action (d) Attraction between opposite charges
- Q2.** A precious stone breaks into four pieces having weights in the proportion 1:2:3:4. The value of such a stone is proportional to the square of its weight. What is the percent loss in the value incurred due to breaking?
- (a) 0 (b) 30 (c) 70 (d) 90
- Q3.** Two runners starting together run on a circular path taking 6 and 8 minutes, respectively, to complete one round. How many minutes later do they meet again for the first time on the start line, assuming constant speeds
- (a) 8 (b) 24 (c) 32 (d) 60
- Q4.** The distribution of grades secured by students in a class is given in the table below.

Grade	Fraction of the Population
A	0.1
B	0.4
C	0.3
D	0.2

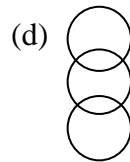
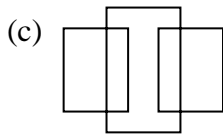
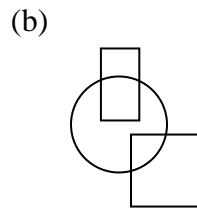
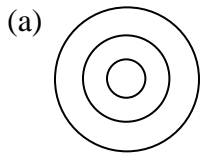
What is the least possible population of the class?

- (a) 2 (b) 4 (c) 8 (d) 10

Q5. The nine numbers $x_1, x_2, x_3, \dots, x_9$, are in ascending order. Their average m is strictly greater than all the first eight numbers. Which of the following is true?

- (a) Average $(x_1, x_2, \dots, x_9, m) > m$ and Average $(x_2, x_3, \dots, x_9) > m$
- (b) Average $(x_1, x_2, \dots, x_9, m) < m$ and Average $(x_2, x_3, \dots, x_9) < m$
- (c) Average $(x_1, x_2, \dots, x_9, m) = m$ and Average $(x_2, x_3, \dots, x_9) > m$
- (d) Average $(x_1, x_2, \dots, x_9, m) < m$ and Average $(x_2, x_3, \dots, x_9) = m$

Q6. Which among the following diagrams represents women, mothers, human beings?



Q7. A boy and a girl make the following statements, of which at most one is correct:

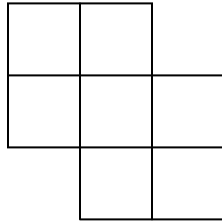
The one in a white shirt says: "I am a girl" (statement – I)

The one in a blue shirt says: "I am a boy" (statement – II)

Which of the following is the correct inference?

- (a) Statement – I is correct but statement – II is incorrect
- (b) Statement – II is correct but statement – I is incorrect
- (c) Both statement I and II are incorrect
- (d) The correctness of the statements I and II cannot be ascertained

Q8. How many quadrilaterals does the following figure have?



- (a) 17 (b) 18 (c) 19 (d) 20

Q9. 12 balls, 3 each of the colours red, green, blue and yellow are put in a box and mixed. If 3 balls are picked at random, without replacement, the probability that all 3 balls are of the same colour is

- (a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{36}$ (d) $\frac{1}{55}$

Q10. Some aliens observe that roosters call before sunrise every day. Having no other information about roosters and sunrises, which of the following inferences would NOT be valid?

- (a) Rooster-call and sunrise may be independent cyclic events with the same periodicity
(b) Both may be triggered by a common cause
(c) Rooster-call may be causing the sunrise
(d) Sunrise cannot be the cause of rooster call as the rooster-call precedes sunrise

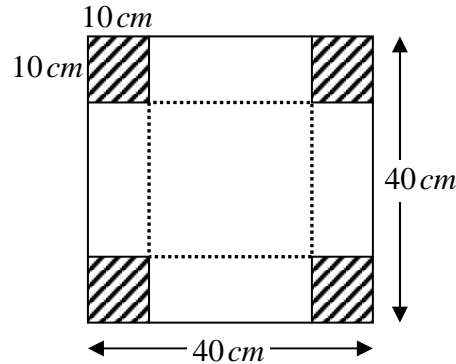
Q11. Twenty-one litres of water in a tank is to be divided into three equal parts using only 5, 8 and 12 litre capacity cans. The minimum number of transfers needed to achieve this is

- (a) 3 (b) 4 (c) 5 (d) 7

Q12. Of four agents Alpha, Beta, Gamma and Delta, three have to be sent together on a mission. If Alpha and Beta cannot go together, Beta and Gamma cannot go together and Gamma and Delta cannot go together, then which of the following holds?

- (a) Any three agents can be sent.
(b) Alpha, Delta and any one out of Beta and Gamma can be sent
(c) Beta, Gamma and any one out of Alpha and Delta can be sent
(d) The mission is impossible.

Q13. An open rectangular box is made by excluding the four identical corners of a piece of paper as shown in the diagram and folding it along the dotted lines



The capacity of the box (in cm^3) is

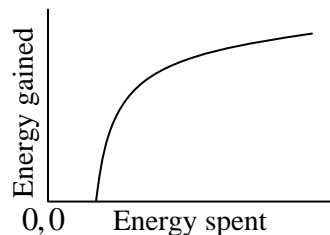
- (a) 8000 (b) 1000 (c) 4000 (d) 6000

Q14. Which of the following is the largest?

$$2^{50}, 3^{40}, 4^{30}, 5^{20}$$

- (a) 2^{50} (b) 3^{40} (c) 4^{30} (d) 5^{20}

Q15. A monkey climbs a tree to eat fruits. The amount of energy gained from eating fruits and the energy spent in climbing on different branches have a relationship shown in the figure.



The ratio of energy gained to energy spent will be the maximum

- (a) At a point where the slope of the curve is the maximum
 (b) At a point where the slope of the curve is unity
 (c) At a point on the curve where the tangent passes through the origin
 (d) At the highest point on the curve

Q16. The length of a cylinder is measured 10 times yielding 10 distinct values. For this set of values, consider the following statements

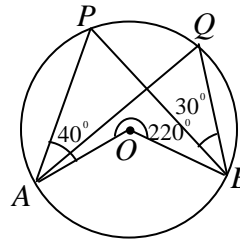
- A. Five of these values will lie above the mean and five below it
- B. Five of these values will lie above median and five below it
- C. At least one value will lie above the mean
- D. At least one value will lie at the median

Which of the statements are necessarily correct?

- (a) B and C (b) A and C (c) B and D (d) A,C and D

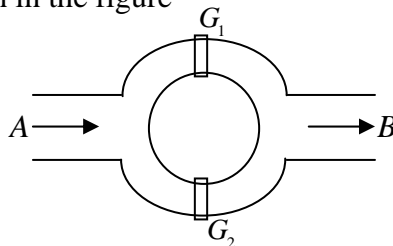
Q17. In the given circle, O is the centre, $\angle PAO = 40^\circ$, $\angle PBQ = 30^\circ$ and outer angle $\angle AOB = 220^\circ$.

Then $\angle AQB$ is



- (a) 70° (b) 80° (c) 60° (d) 110°

Q18. A canal system is shown in the figure



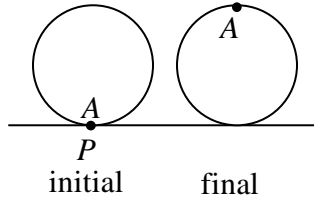
Water flows from A to B through two channels. Gates G_1 and G_2 , are operated independently to regulate the flow. Probability of G_1 to be open is 10% while that of G_2 is 20%. The probability that water will flow from A to B is

- (a) 10% (b) 20% (c) 28% (d) 30%

Q19. A long ream of paper of thickness t is rolled tightly. As the roll becomes larger, the length of the paper wrapped in one turn exceeds the length in the previous turn by

- (a) t (b) $2t$ (c) πt (d) $2\pi t$

Q20. Point A on a wheel of radius r touches the horizontal plane at point P . It rolls without slipping, till point A is at the highest position in the first turn. What is the final distance AP ?

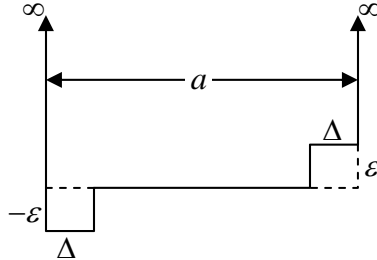


- (a) $2r$ (b) $r\sqrt{(1+\pi^2)}$ (c) $r\sqrt{(4+\pi^2)}$ (d) $2r\sqrt{(1+\pi^2)}$

PART- B

- Q21.** An object is dropped on a cushion from a height $10m$ above it. On being hit, the cushion is depressed by $0.1 m$. Assuming that the cushion provides a constant resistive force, the deceleration of the object after hitting the cushion, in terms of the acceleration due to gravity g is
- (a) $10 g$ (b) $50 g$ (c) $100 g$ (d) g
- Q22.** A turn-table is rotating with a constant angular velocity ω_0 . In the rotating frame fixed to the turntable, a particle moves radially outwards at a constant speed v_0 . The acceleration of the particle in the $r\theta$ coordinates, as seen from an inertial frame, the origin of which is at the centre of the turntable, is
- (a) $-r\omega_0^2\hat{r}$ (b) $2r\omega_0^2\hat{r} + v_0\omega_0\hat{\theta}$
(c) $r\omega_0^2\hat{r} + 2v_0\omega_0\hat{\theta}$ (d) $-r\omega_0^2\hat{r} + 2v_0\omega_0\hat{\theta}$
- Q23.** Assume that the earth revolves in a circular orbit around the sun. Suppose the gravitational constant G varies slowly as a function of time. In particular, it decreases to half its initial value in the course of one million years. Then during this time the
- (a) Radius of the earth's orbit will increase by a factor of two
(b) Total energy of the earth remains constant
(c) Orbital angular momentum of the earth will increase
(d) Radius of the earth's orbit remains the same.
- Q24.** A particle of mass m moves in One dimension in the potential $V(x) = kx^4, (k > 0)$. At time $t = 0$ the particle starts from rest at $x = A$. For bounded motion, the time period of its motion is
- (a) Proportional to $A^{-1/2}$
(b) Proportional to A^{-1}
(c) Independent of A
(d) Not well-defined (the system is chaotic)

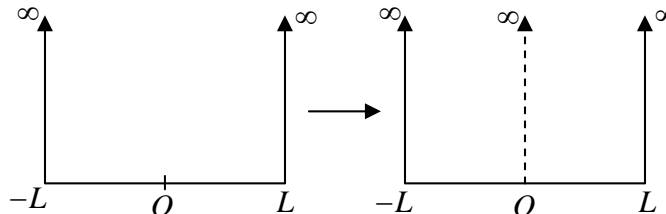
Q25. The infinite square-well potential of a particle in a box of size a is modified as shown in the figure below (assume $\Delta \ll a$)



The energy of the ground state, compared to the ground state energy before the perturbation was added

- (a) Increases by a term of order ϵ (b) Decreases by a term of order ϵ
 (c) Increases by a term of order ϵ^2 (d) Decreases by a term of order ϵ^2

Q26. A quantum particle of mass m in one dimension, confined to a rigid box as shown in the figure, is in its ground state. An infinitesimally thin wall is very slowly raised to infinity at the centre of the box, in such a way that the system remains in its ground state at all times. Assuming that no energy is lost in raising the wall, the work done on the system when the wall is fully raised, eventually separating the original box into two compartments, is



- (a) $\frac{3\pi^2\hbar^2}{8mL^2}$ (b) $\frac{\pi^2\hbar^2}{8mL^2}$ (c) $\frac{\pi^2\hbar^2}{2mL^2}$ (d) 0

Q27. The wavefunction of a free particle of mass m , constrained to move in the interval $-L \leq x \leq L$, is $\psi(x) = A(L+x)(L-x)$, where A is the normalization constant. The

probability that the particle will be found to have the energy $\frac{\pi^2\hbar^2}{2mL^2}$ is

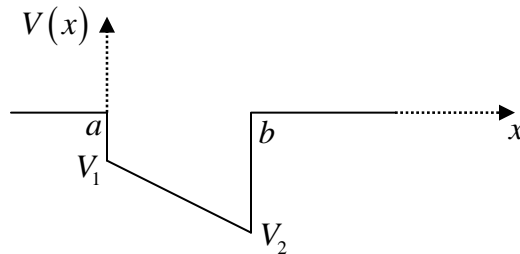
- (a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{\pi}$

Q28. A particle moving in a central potential is described by a wavefunction $\psi(r) = zf(r)$ where $r = (x, y, z)$ is the position vector of the particle and $f(r)$ is a function of $r = |r|$.

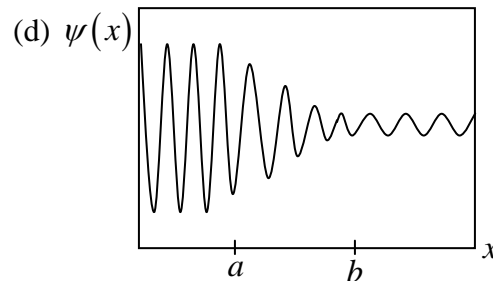
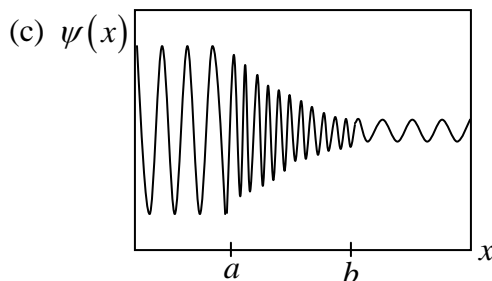
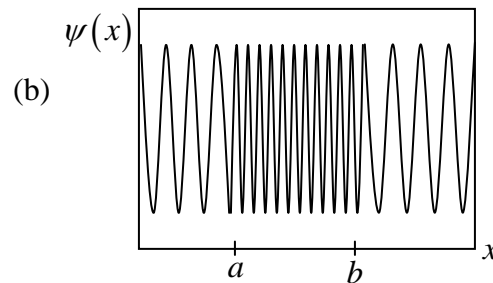
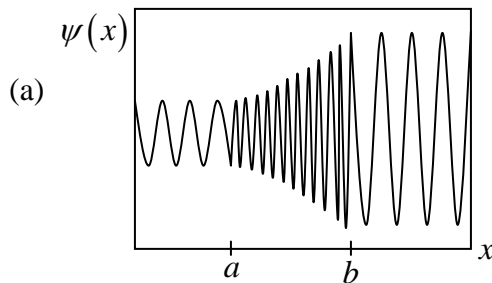
If L is the total angular momentum of the particle, the value of L^2 must be

- (a) $2\hbar^2$ (b) \hbar^2 (c) $4\hbar^2$ (d) $\frac{3}{4}\hbar^2$

Q29. A particle of mass m and energy $E > 0$ in one dimension is scattered by the potential below.



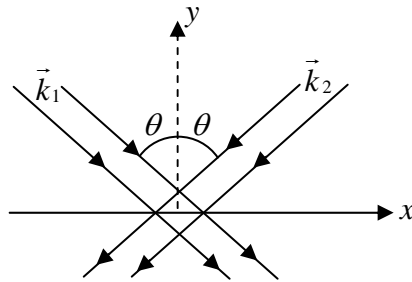
If the particle was moving from $x = -\infty$ to $x = \infty$, which of the following graphs gives the best qualitative representation of the wavefunction of this particle?



Q30. Consider a planar wire loop as an n -sided regular polygon, in which R is the distance from the centre to a vertex. If a steady current I flows through the wire, the magnitude of the magnetic field at the centre of the Loop is

- (a) $\frac{\mu_0 I}{2R} \sin\left(\frac{2\pi}{n}\right)$ (b) $\frac{\mu_0 n I}{4\pi R} \sin\left(\frac{\pi}{n}\right)$ (c) $\frac{\mu_0 n I}{2\pi R} \tan\left(\frac{2\pi}{n}\right)$ (d) $\frac{\mu_0 n I}{2\pi R} \tan\left(\frac{\pi}{n}\right)$

Q31. Two coherent plane electromagnetic waves of wavelength $0.5 \mu\text{m}$ (both have the same amplitude and are linearly polarized along the z -direction) fall on the $y=0$ plane. Their wave vectors \vec{k}_1 and \vec{k}_2 are as shown in the figure



If the angle θ is 30° , the fringe spacing of the interference pattern produced on the plane is

- (a) $1.0 \mu\text{m}$ (b) $0.29 \mu\text{m}$ (c) $0.58 \mu\text{m}$ (d) $0.5 \mu\text{m}$

Q32. Which of the following is **not** a correct boundary condition at an interface between two homogeneous dielectric media? (In the following \hat{n} is a unit vector normal to the interface, σ and \vec{j}_s , are the surface charge and current densities, respectively.)

- (a) $\hat{n} \times (\vec{D}_1 - \vec{D}_2) = 0$ (b) $\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{j}_s$
 (c) $\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \sigma$ (d) $\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

Q33. The permittivity tensor of a uniaxial anisotropic medium, in the standard Cartesian basis,

is $\begin{pmatrix} 4\varepsilon_0 & 0 & 0 \\ 0 & 4\varepsilon_0 & 0 \\ 0 & 0 & 9\varepsilon_0 \end{pmatrix}$ where ε_0 is a constant. The wave number of an electromagnetic

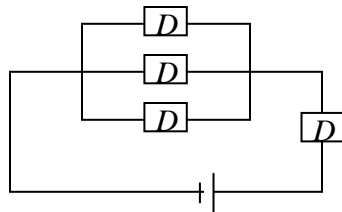
plane wave polarized along the x -direction, and propagating along the y -direction in this medium (in terms of the wave number k_0 of the wave in vacuum) is

- (a) $4k_0$ (b) $2k_0$ (c) $9k_0$ (d) $3k_0$

Q34. The element of a 3×3 matrix A are the products of its row and column indices $A_{ij} = ij$ (where $i, j = 1, 2, 3$). The eigenvalues of A are

- (a) $(7, 7, 0)$ (b) $(7, 4, 3)$ (c) $(14, 0, 0)$ (d) $\left(\frac{14}{3}, \frac{14}{3}, \frac{14}{3}\right)$

Q35. In the following circuit, each device D may be an insulator with probability p or a conductor with probability $(1-p)$.



The probability that a non-zero current flows through the circuit is

- (a) $2 - p - p^3$ (b) $(1-p)^4$
 (c) $(1-p)^2 p^2$ (d) $(1-p)(1-p^3)$

Q36. The solution of the differential equation $x \frac{dy}{dx} + (1+x)y = e^{-x}$ with the boundary condition $y(x=1) = 0$, is

- (a) $\frac{(x-1)}{x} e^{-x}$ (b) $\frac{(x-1)}{x^2} e^{-x}$
 (c) $\frac{(1-x)}{x^2} e^{-x}$ (d) $(x-1)^2 e^{-x}$

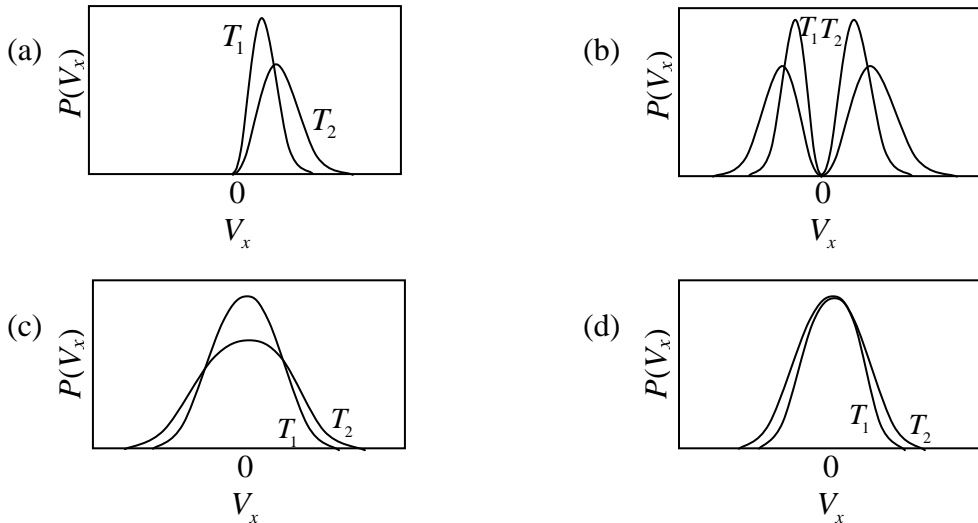
Q37. The value of the definite integral $\int_0^\pi \frac{d\theta}{5+4\cos\theta}$ is

- (a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{3}$

Q38. In a system comprising of approximately 10^{23} distinguishable particles, each particle may occupy any of 20 distinct states. The maximum value of the entropy per particle is nearest to

- (a) $20k_B$ (b) $3k_B$ (c) $10(\ln 2)k_B$ (d) $20(\ln 2)k_B$

- Q39.** Consider a classical gas in thermal equilibrium at temperatures T_1 and T_2 where $T_1 < T_2$. Which of the following graphs correctly represents the qualitative behaviour of the probability density function of the x -component of the velocity?



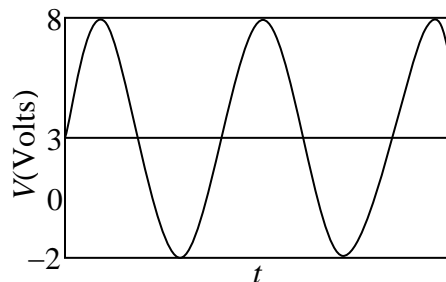
- Q40.** The equation of state of an ideal gas is $pV = RT$. At very low temperatures, the volume expansion coefficient $\frac{1}{V} \frac{\partial V}{\partial T}$ at constant pressure
- (a) Diverges as $\frac{1}{T^2}$ (b) Diverges as $\frac{1}{T}$
- (c) Vanishes as T (d) Is independent of the temperature

- Q41.** The Hamiltonian of a classical nonlinear one dimensional oscillator is $H = \frac{1}{2m} p^2 + \lambda x^4$, where $\lambda > 0$ is a constant. The specific heat of a collection of a collection of N independent such oscillators is
- (a) $\frac{3Nk_B}{2}$ (b) $\frac{3Nk_B}{4}$ (c) Nk_B (d) $\frac{Nk_B}{2}$

Q42. In an experiment to measure the acceleration due to gravity g using a simple pendulum, the length and time period of the pendulum are measured to three significant figures. The mean value of g and the uncertainty δg of the measurements are then estimated using a calculator from a large number of measurements and found to be $9.82147 m/s^2$ and $0.02357 m/s^2$, respectively. Which of the following is the most accurate way of presenting the experimentally determined value of g ?

- (a) $9.82 \pm 0.02 m/s^2$ (b) $9.8215 \pm 0.02 m/s^2$
 (c) $9.82147 \pm 0.02357 m/s^2$ (d) $9.82 \pm 0.02357 m/s^2$

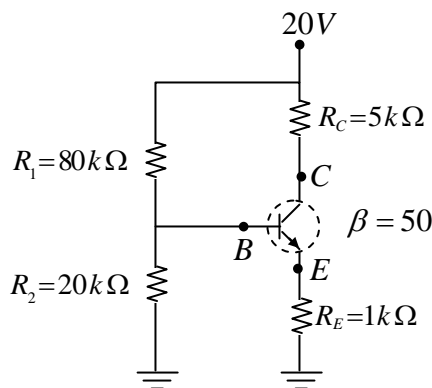
Q43. An ac signal of the type as shown in the figure, is applied across a resistor $R = 1 \Omega$.



The power dissipated across the resistor is

- (a) $12.5 W$ (b) $9 W$ (c) $25 W$ (d) $21.5 W$

Q44. An *npn*-transistor is connected in a voltage divider configuration as shown in the figure below.

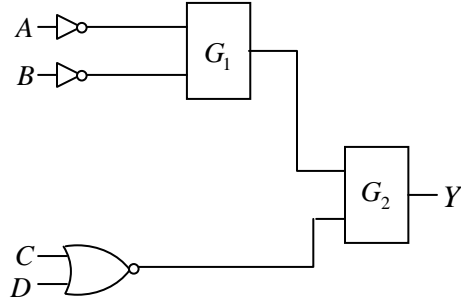


If the resistor R_2 is disconnected, the voltages V_B at the base and V_C at the collector change as follows.

- (a) both V_B and V_C increase (b) both V_B and V_C decrease
 (c) V_B decreases, but V_C increases (d) V_B increases, but V_C decreases

PART - C

Q45. Let Y denote the output in the following logical Circuit.

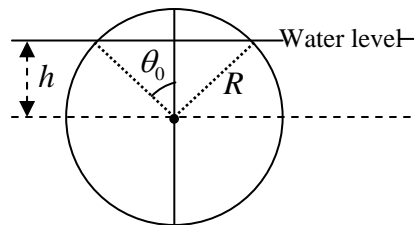


If $Y = AB + \overline{CD}$, the gates G_1 and G_2 must, respectively, be

- (a) OR and NAND (b) NOR and OR
 (c) AND and NAND (d) NAND and OR

Q46. A solid spherical Cork of radius R and specific gravity 0.5 floats on water. The cork is pushed down so that its Centre of mass is at a distance h (where $0 < h < R$) below the surface of water, and Then released. The volume of the part of the cork above water level

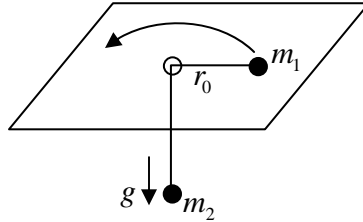
is $\pi R^3 \left(\frac{2}{3} - \cos \theta_0 + \frac{1}{3} \cos^3 \theta_0 \right)$ where θ_0 is the angle as shown in the figure.



At the moment of release, the dependence of the upward force on the cork on h is

- (a) $\frac{h}{R} - \frac{1}{3} \left(\frac{h}{R} \right)^3$ (b) $\frac{h}{R} + \frac{1}{3} \left(\frac{h}{R} \right)^3$ (c) $\frac{h}{R} - \frac{2}{3} \left(\frac{h}{R} \right)^3$ (d) $\frac{h}{R} + \frac{2}{3} \left(\frac{h}{R} \right)^3$

- Q47.** Two particles of masses m_1 and m_2 are connected by a massless thread of length l as shown in figure below.



The particle of mass m_1 on the plane undergoes a circular motion with radius r_0 and angular momentum L . When a small radial displacement ϵ (where $\epsilon \ll r_0$) is applied, its radial coordinate is found to oscillate about r_0 . The frequency of the oscillations is

- (a) $\sqrt{\frac{7m_2g}{\left(m_1 + \frac{m_2}{2}\right)r_0}}$ (b) $\sqrt{\frac{7m_2g}{(m_1 + m_2)r_0}}$ (c) $\sqrt{\frac{3m_2g}{\left(m_1 + \frac{m_2}{2}\right)r_0}}$ (d) $\sqrt{\frac{3m_2g}{(m_1 + m_2)r_0}}$

- Q48.** The time evolution of a coordinate x of a particle is described by the equation

$$\frac{d^2x}{dt^4} + 2\Omega^2 \frac{d^2x}{dt^2} + (\Omega^4 - A^4)x = 0$$

For $\Omega > A$, the particle will

- (a) Eventually come to rest at the origin
 (b) Eventually drift to infinity ($|x| \rightarrow \infty$)
 (c) Oscillate about the origin
 (d) Eventually come to rest at $\frac{\Omega}{A}$ or $-\frac{\Omega}{A}$
- Q49.** The Hamiltonian of a quantum particle of mass m is $H = \frac{p^2}{2m} + \alpha|x|^r$, where α and r are positive constants. The energy E_n of the n^{th} level for large n , depends on n as
- (a) n^{2r} (b) n^{r+2} (c) $n^{1/(r+2)}$ (d) $n^{2r/(r+2)}$

Q50. In the partial wave expansion, the differential scattering cross-section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \left| \sum_l (2l+1) e^{i\delta_l} \sin\delta_l P_l(\cos\theta) \right|^2$$

where θ is the scattering angle. For a certain neutron-nucleus scattering. It is found that the two lowest phase shifts δ_0 and δ_1 corresponding to s -wave and p -wave, respectively, satisfy $\delta_1 \approx \frac{\delta_0}{2}$. Assuming that the other phase shifts are negligibly small, the differential cross-section reaches its minimum for $\cos\theta$ equal to

- (a) 0 (b) ± 1 (c) $-\frac{2}{3}\cos^2\delta_1$ (d) $\frac{1}{3}\cos^2\delta_1$

Q51. A charged, spin-less particle of mass m is subjected to an attractive potential $V(x, y, z) = \frac{1}{2}k(x^2 + y^2 + z^2)$, where k is a positive constant. Now a perturbation in the form of a weak magnetic field $B = B_0\hat{k}$ (where B_0 is a constant) is switched on. Into how many distinct levels will the second excited state of the unperturbed Hamiltonian split?

- (a) 5 (b) 4 (c) 2 (d) 1

Q52. The elastic scattering of a charged particle of mass m off an atom can be approximated by the potential $V(r) = \frac{\alpha}{r}e^{-r/R}$ where α and R are positive constants.

If the wave number of the incoming particle is k and the scattering angle is 2θ , the differential cross-section in the Born approximation is

- (a) $\frac{m^2\alpha^2R^4}{4\hbar^4(1+k^3R^2\sin^2\theta)}$ (b) $\frac{m^2\alpha^2R^4}{\hbar^4(2k^2R^2\sin^2\theta)^2}$
 (c) $\frac{2m^2\alpha^2R^4}{\hbar^4(2k^2R^2\sin^2\theta)}$ (d) $\frac{4m^2\alpha^2R^4}{\hbar^4(1+4k^2R^2\sin^2\theta)^2}$

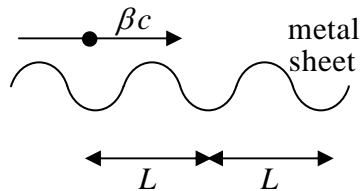
Q53. The wave number k and the angular frequency ω of a wave are related by the dispersion relation $\omega^2 = \alpha k + \beta k^3$ where α and β are positive constants. The wave number for which the phase velocity equals the group velocity, is

- (a) $3\sqrt{\frac{\alpha}{\beta}}$ (b) $\sqrt{\frac{\alpha}{\beta}}$ (c) $\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}$ (d) $\frac{1}{3}\sqrt{\frac{\alpha}{\beta}}$

Q54. An inertial observer A at rest measures the electric and magnetic field $E = (\alpha, 0, 0)$ and $B = (\alpha, 0, 2\alpha)$ in a region, where α is a constant. Another inertial observer B , moving with a constant velocity with respect to A , measures the fields as $E' = (E'_x, \alpha, 0)$ and $B' = (\alpha, B'_y, \alpha)$. Then in units $c = 1$, E'_x and B'_y are given, respectively, by

- (a) -2α and α (b) 2α and $-\alpha$
 (c) α and -2α (d) $-\alpha$ and 2α

Q55. A point charge is moving with a uniform velocity βc along the positive x -direction, parallel to and very close to a corrugated metal sheet (see the figure below).



The wavelength of the electromagnetic radiation received by an observer along the direction of motion is

- (a) $\frac{1}{\beta}\sqrt{1-\beta^2}$ (b) $L\sqrt{1-\beta^2}$ (c) $L\beta\sqrt{1-\beta^2}$ (d) L

Q56. If the Newton-Raphson method is used to find the positive root of the equation $x = 2 \sin x$, the iteration equation is

- (a) $x_{n+1} = \frac{2x_n - 2(\sin x_n + x_n \cos x_n)}{1 - 2 \cos x_n}$ (b) $x_{n+1} = \frac{2(\sin x_n - x_n \cos x_n)}{1 - 2 \cos x_n}$
 (c) $x_{n+1} = \frac{x_n^2 - 1 + 2(\cos x_n - x_n \sin x_n)}{x_n - 2 \sin x_n}$ (d) $x_{n+1} = \frac{x_n^2 - 1 - 2(\cos x_n + \sin x_n)}{x_n - 2 \sin x_n}$

- Q57.** The equation of motion of a forced simple harmonic oscillator is $\ddot{x} + \omega^2 x = A \cos \Omega t$, where A is a constant. At resonance $\Omega = \omega$ the amplitude of oscillations at large times
- (a) saturates to a finite value (b) increases with time as \sqrt{t}
 (c) increases linearly with time (d) increases exponentially with time
- Q58.** The operator A has a matrix representation $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ in the basis spanned by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In another basis spanned by $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, the matrix representation of A is
- (a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$
- Q59.** The operator $x \frac{d}{dx} \delta(x)$, where $\delta(x)$ is the Dirac delta function, acts on the space of real-valued square-integrable functions on the real line. This operator is equivalent to
- (a) $-\delta(x)$ (b) $\delta(x)$ (c) x (d) 0
- Q60.** At each time step, a random walker in one dimension either remains at the same point with probability $\frac{1}{4}$, or moves by a distance Δ to the right or left with probabilities $\frac{3}{8}$ each. After N time steps, its root mean squared displacement is
- (a) $\Delta\sqrt{N}$ (b) $\Delta\sqrt{\frac{9N}{16}}$ (c) $\Delta\sqrt{\frac{3N}{4}}$ (d) $\Delta\sqrt{\frac{3N}{8}}$
- Q61.** The Hamiltonian of three Ising spins S_1, S_2 and S_3 , each taking values ± 1 , is $H = -J(S_1 S_2 + S_2 S_3) - h S_1$, where J and h are positive constants. The mean value of S_3 in equilibrium at a temperature $T = 1/(k_B \beta)$, is
- (a) $\tanh^3(\beta J)$ (b) $\tan(\beta h) \tanh^2(\beta J)$
 (c) $\sinh(\beta h) \sinh^2(\beta J)$ (d) 0

Q62. The free energy of a magnetic system, as a function of its magnetisation m , is

$$F = \frac{1}{2}am^2 - \frac{1}{4}bm^4 + \frac{1}{6}m^6. \text{ where } a \text{ and } b \text{ are positive constants.}$$

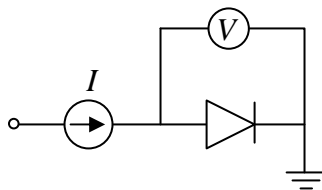
At a fixed value of a , the critical value of b , above which the minimum of F will be at a non-zero value of magnetisation, is

- (a) $\sqrt{\frac{10a}{3}}$ (b) $\sqrt{\frac{16a}{3}}$ (c) $\frac{10}{3}\sqrt{a}$ (d) $\frac{16}{3}\sqrt{a}$

Q63. For optimal performance of an op-amp based current-to-voltage converter circuit, the input and output impedance should be

- (a) Low input impedance and high output impedance
 (b) Low input impedance and low output impedance
 (c) High input impedance and high output impedance
 (d) High input impedance and low output impedance

Q64. The forward diode current is given by $I = kT^\alpha e^{-E_g/k_B T} (\exp(eV/k_B T) - 1)$, where E_g is the band gap of the semiconductor, V is the voltage drop across the diode, T is the temperature of the diode operating near room temperature and, α and K are constants. A diode is used as a thermal sensor in the circuit shown below.



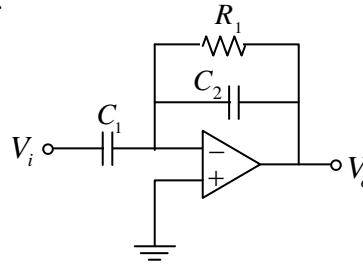
If V is measured using an ideal voltmeter to estimate T , the variation of the voltage V as a function of T is best approximated by (in the following a and b are constants)

- (a) $aT^2 + b$ (b) $aT + b$ (c) $aT^3 + b$ (d) $aT + bT^2$

Q65. A circuit constructed using op-amp, resistor $R_1 = 1\text{ k}\Omega$ and capacitors $C_1 = 1\text{ }\mu\text{F}$ and $C_2 = 0.1\text{ }\mu\text{F}$ is shown in the figure below.

This circuit will act as a

- (a) High pass filter
- (b) Low pass filter
- (c) Band pass filter
- (d) Band reject filter



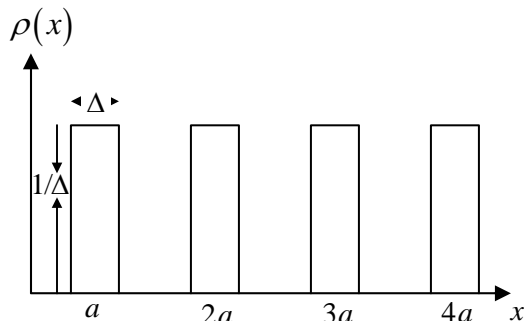
Q66. The third-nearest neighbour distance in a BCC (Body Centered Cubic) crystal with lattice constant a_0 is

- (a) a_0
- (b) $\frac{3a_0}{2}$
- (c) $\sqrt{3}a_0$
- (d) $\sqrt{2}a_0$

Q67. A bound electron and hole pair interacting via Coulomb interaction in a semiconductor is called an exciton. The effective masses of an electron and a hole are about $0.1m_e$ and $0.5m_e$ respectively, where m_e is the rest mass of the electron. The dielectric constant of the semiconductor is 10. Assuming that the energy levels of the excitons are hydrogen-like, the binding energy of an exciton (in units of the Rydberg constant) is closest to

- (a) 2×10^{-3}
- (b) 2×10^{-4}
- (c) 8×10^{-4}
- (d) 3×10^{-3}

Q68. Consider an array of atoms in one dimension with an ensemble averaged periodic density distribution as shown in the figure.



If k is the wave number and $S(k, \Delta)$ denotes the Fourier transform of the density-density correlation function, the ratio $\frac{S(k, \Delta)}{S(k, 0)}$ is

- | | |
|--|--|
| (a) $\cos\left(\frac{k\Delta}{2}\right)$ | (b) $\cos^2\left(\frac{k\Delta}{2}\right)$ |
| (c) $\frac{2}{k\Delta} \sin\left(\frac{k\Delta}{2}\right)$ | (d) $\frac{4}{k^2\Delta^2} \sin^2\left(\frac{k\Delta}{2}\right)$ |

Q69. A doubly charged ion in the angular momentum state ($J = 2, J_3 = 1$) meets a gas of polarized electrons $\left(S_3 = \frac{1}{2}\right)$ and gets neutralized. If the orbital angular momentum transferred in the process is zero, the probability that the neutral atom is in the ($J = 2, J_3 = 2$) state is

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| (a) $\frac{2}{5}$ | (b) $\frac{2}{3}$ | (c) $\frac{1}{5}$ | (d) $\frac{1}{3}$ |
|-------------------|-------------------|-------------------|-------------------|

Q70. The range of the inter-atomic potential in gaseous hydrogen is approximately 5 \AA . In thermal equilibrium, the maximum temperature for which the atom-atom scattering is dominantly s -wave, is

- | | | | |
|---------------------|---------------------|-------------------|--------------------|
| (a) 500 K | (b) 100 K | (c) 1 K | (d) 1 mK |
|---------------------|---------------------|-------------------|--------------------|

Q71. The energy levels corresponding to the rotational motion of a molecule are $E_J = BJ(J+1)cm^{-1}$ where $J = 0, 1, 2, \dots$, and B is a constant. Pure rotational Raman transitions follow the selection rule $\Delta J = 0, \pm 2$. When the molecule is irradiated, the separation between the closest Stokes and anti-Stokes lines (in cm^{-1}) is

- (a) $6B$ (b) $12B$ (c) $4B$ (d) $8B$

Q72. The cavity of a He-Ne laser emitting at $632.8nm$, consists of two mirrors separated by a distance of $35cm$. If the oscillations in the laser cavity occur at frequencies within the gain bandwidth of $1.3GHz$, the number of longitudinal modes allowed in the cavity is

- (a) 1 (b) 2 (c) 3 (d) 4

Q73. An excited state of a 8_4Be nucleus decays into two α -particles which are in a spin-parity 0^+ state. If the mean life-time of this decay is $10^{-22}s$, the spin-parity of the excited state of the nucleus is

- (a) 2^+ (b) 3^+ (c) 0^- (d) 4^-

Q74. The elastic scattering of a neutrino ν_e by an electron e^- , i.e. the reaction $\nu_e + e^- \rightarrow \nu_e + e^-$ can be described by the interaction Hamiltonian

$$H_{int} = \frac{1}{\sqrt{2}} G_F \int d^3x (\bar{\psi}_e(x) \gamma^\mu \psi_{\nu_e}(x)) (\bar{\psi}_{\nu_e}(x) \gamma_\mu \psi_e(x))$$

The cross-section of the above process depends on the centre of mass energy E , as

- (a) $\frac{1}{E^2}$ (b) E^2 (c) E (d) \sqrt{E}

Q75. The mean life-time of the following decays:

$\rho_0 \rightarrow \pi^+ + \pi^-$, $\pi^0 \rightarrow \gamma + \gamma$, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$, are τ_ρ , τ_π and τ_μ respectively.

They satisfy

- (a) $\tau_\pi < \tau_\rho < \tau_\mu$ (b) $\tau_\mu < \tau_\rho < \tau_\pi$
 (c) $\tau_\rho < \tau_\pi < \tau_\mu$ (d) $\tau_\rho < \tau_\mu < \tau_\pi$