NET December 2017

PART A

Q1. A leaf appears green in daylight. If this leaf were observed in red light, what colour would it appear to have?

(a) Green   (b) Black-Brown   (c) Red   (d) Blue

Q2. The distance from Nehrunagar to Gandhinagar is $27\,km$. A and B start walking from Nehrunagar towards Gandhinagar at speeds of $5\,km/hr$ and $7\,km/hr$, respectively. B reaches Gandhinagar, returns immediately, and meets A at Indiranagar. What is the distance between Nehrunagar and Indiranagar? (Assume all three cities to be in one straight line)

(a) $12.5\,km$ (b) $22.5\,km$ (c) $4.5\,km$ (d) $13.5\,km$

Q3. A sphere G of radius b is fixed mid-air and several spheres identical to the first one are shot at it with their velocities parallel to each other. If the shot spheres fall within an imaginary cylinder of radius $a\,(b << a)$, then the fraction of spheres that will hit G is

(a) $2b/a$ (b) $4b^2/a^2$ (c) $(a-b)/(a+b)$ (d) $8b^3/a^3$

Q4. Five persons A, B, C, D and E are sifting in a row with C in the middle of the group. If D is at an extreme end and there are at least two persons between B and E, then which of the following statements is incorrect?

(a) E can be on extreme left   (b) E can be on extreme right

(c) A cannot be on extreme left   (d) A is always a neighbour of B or D

Q5. In a group of students, 30% play only cricket, 20 % play only football and 10 % play only basketball. 20 % of the students play both football and cricket, 15 % play both basketball and cricket, 10 % play both football and basketball. 15 students play no games, while 5 % of the students play all three games. What is the total number of students?

(a) 300   (b) 250   (c) 350   (d) 400

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Q6.   When Ramesh was at the age of 8 years, he hammered a nail into a large tree to mark his height. If the tree grows 2 cm/year, how much higher would the nail be after 5 years?
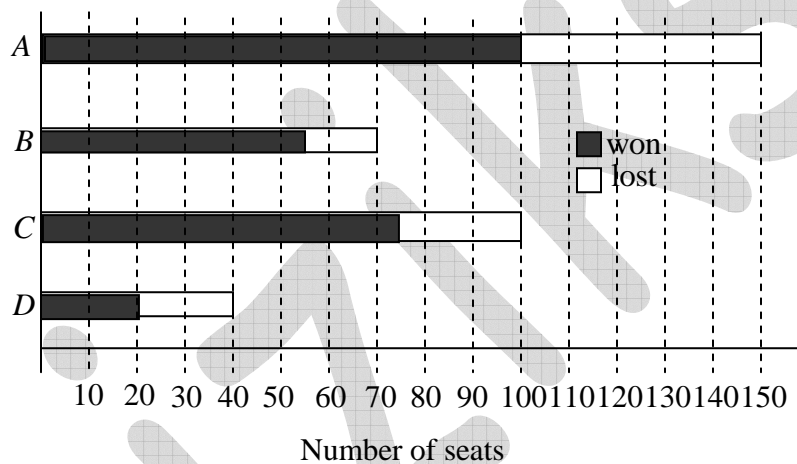
(a) $5\,cm$ higher   (b) $0\,cm$ higher   (c) $10\,cm$ higher   (d) $8\,cm$ higher

Q7.   Find the missing number

| 17 | 15 | | 13 | 12 | | 25 | 24 | | 41 | 40 |
| 8 | | | 5 | | | 7 | | | ? | |

(a) 4   (b) 9   (c) 3   (d) 6

Q8.

Number of seats

The bar chart above shows number of seats won by four political parties A, B, C and D. Which party won the largest proportion of seats it contested?

(a) A (b) B (c) C (d) D

Q9.   The molar fraction of hydrochloric acid in an extremely dilute' aqueous solution is doubled. The pH of the resulting solution is

(a) approximately doubled   (b) approximately halved

(c) increased   (d) reduced

Q10.   Approximately how much blood flows per day through a normal human heart beating 70 times per minute, having a relaxed volume of 110 cc and compressed volume of 70 cc?

(a) 7150 litres   (b) 4000 litres   (c) 28000 litres   (d) 11100 litres

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Q11. The number of three English letter words, having at least one consonant, but not having two consecutive consonants, is

(a) 2205   (b) 3780   (c) 2730   (d) 3360

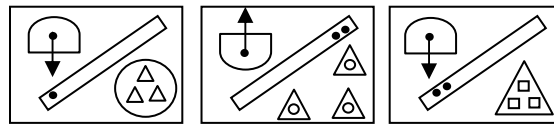Q12. Which one of the following graphs represents $f(x) = \sin x \cos x$?

(a)

(b)

(c)

(d)
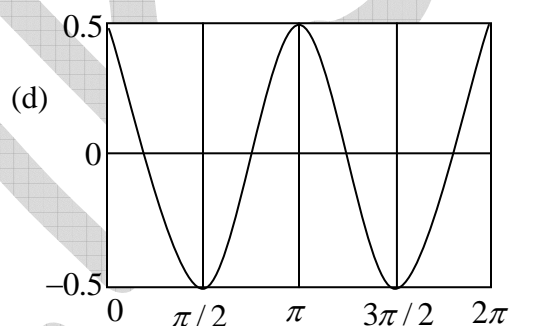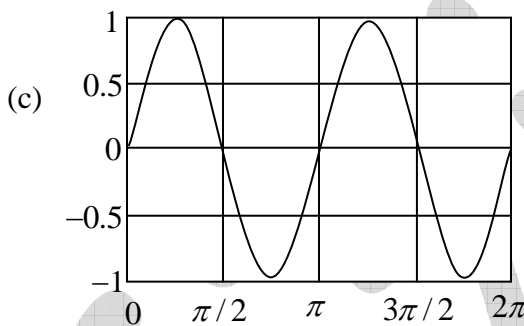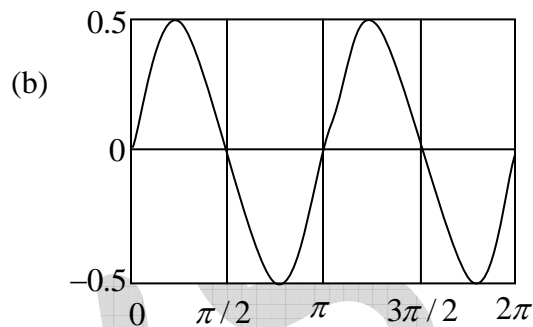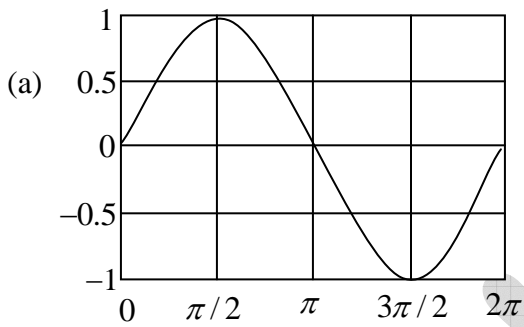
Q13. There are two gas parcels of equal volume, A and B at the same temperature and pressure. Parcel A is one mole of water vapour, while parcel B is one mole of dry air. Which of the following is TRUE?

(a) Parcel A is heavier then Parcel B

(b) Parcel B is heavier than Parcel A

(c) Both parcels are equally heavy

(d) Without temperature and pressure data, their relative masses cannot be determined

Q14. For which of the following numbers is its positive square root closest to the number itself'?

(a) 0.33   (b) 0.99   (c) 0.89   (d) 0.10

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Q15. Find the next pattern in the following sequence:

(a)

(b)

(c)

(d)

Q16. DRQP is a small square of side a in the corner of a big square ABCD of side A. What is the ratio of the area of the quadrilateral *PBRQ* to that of the square *ABCD*, given $A/a = 3$?
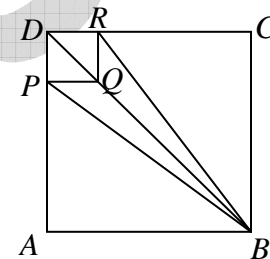
(a) $2/9$

(b) $1/6$

(c) $1/3$

(d) $2/7$

Q17. A 100 m long fence is to be made by fixing a wire mesh on steel poles. Each pole has a 1 m vertical portion and a 1 m portion tilted at 45^0 to the vertical. What will be the area of wire mesh required?

(a) 200 m^2 (b) 241.4 m^2 (c) 400 m^2 (d) 170.7 m^2

Q18. The average staff salary of a company is Rs. $8000/-$. A new guard and a new manager are recruited with salaries of Rs. $5000/-$ and $20000/-$, respectively. What is the current staff strength if the new average salary is Rs. $4000/-$ more than that of the guard?

(a) 7   (b) 9   (c) 10   (d) 11

Q19. A bird flies along the three sides of a field in the shape of an equilateral triangle at speeds of $2, 4, 8$ km/hr, respectively. The average speed of the bird is

(a) $\dfrac{24}{7}$ km/hr (b) $\dfrac{14}{3}$ km/hr (c) $\dfrac{22}{7}$ km/hr (d) 4 km/hr

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Q20. A buys n copies of a book at 20 % discount. B gets the same book at 30 % discount. What is the minimum value of it for which B can buy one extra copy of the book, spending the same amount as A ?

(a) 7

(b) 8

(c) 6

(d) This problem cannot be solved unless the marked price of the book is known.

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PART 'B'

Q21. Let A be a non-singular 3×3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $\left(A^T\right)^{-1}$. Which of the following is true?

(a) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 1$

(b) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 1, \vec{u} \cdot \vec{c} = 0$

(c) $\vec{u} \cdot \vec{a} = 1, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$

(d) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$

Q22. Consider the real function $f(x) = 1/(x^2 + 4)$. The Taylor expansion of $f(x)$ about $x = 0$ converges

(a) for all values of x

(b) for all values of x except $x = \pm 2$

(c) in the region $-2 < x < 2$

(d) for $x > 2$ and $x < -2$

Q23. Consider the matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution and the corresponding normalised solution (upto a sign) is

(a) $b = 2c$ and $(x, y, z) = \dfrac{1}{\sqrt{6}}(1, -2, 1)$

(b) $c = 2b$ and $(x, y, z) = \dfrac{1}{\sqrt{6}}(1, 1, -2)$

(c) $c = b + 1$ and $(x, y, z) = \dfrac{1}{\sqrt{6}}(2, -1, -1)$

(d) $b = c + 1$ and $(x, y, z) = \dfrac{1}{\sqrt{6}}(1, -2, 1)$

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Q24. Consider the differential equation $\dfrac{dy}{dt} + ay = e^{-bt}$ with the initial condition $y(0) = 0$.

Then the Laplace transform $Y(s)$ of the solution $y(t)$ is

(a) $\dfrac{1}{(s+a)(s+b)}$ (b) $\dfrac{1}{b(s+a)}$ (c) $\dfrac{1}{a(s+b)}$ (d) $\dfrac{e^{-a} - e^{-b}}{b-a}$

Q25. A light signal travels from a point A to a point B, both within a glass slab that is moving with uniform velocity (in the same direction as the light) with speed $0.3c$ with respect to an external observer. If the refractive index of the slab is 1.5, then the observer will measure the speed of the signal as

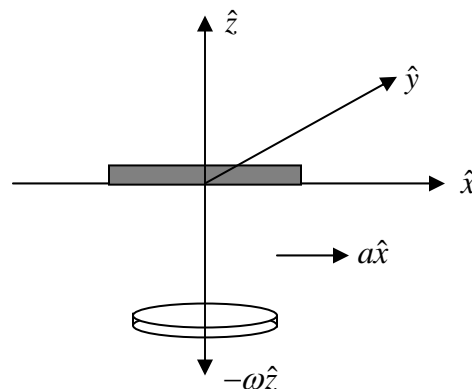(a) $0.67c$ (b) $0.81c$ (c) $0.97c$ (d) c

Q26. A monoatomic gas 'of volume V is in equilibrium in a uniform vertical cylinder, the lower end of which is closed by a rigid wall and the other by a frictionless piston. The piston is pressed lightly and released. Assume that the gas is a poor conductor of heat and the cylinder and piston are perfectly insulating. If the cross-sectional area of the cylinder is A, the angular frequency of small oscillations of the piston about the point of equilibrium, is

(a) $\sqrt{5gA/(3V)}$ (b) $\sqrt{4gA/(3V)}$ (c) $\dfrac{5}{3}\sqrt{gA/V}$ (d) $\sqrt{7gA/(5V)}$

Q27. A disc of mass m is free to rotate in a plane parallel to the xy plane with an angular velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.

The car accelerates in the positive x-direction with an acceleration $a > 0$. Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?

(a) only the x and the z coordinates change

(b) only the y and the z coordinates change

(c) only the x and the y coordinates change

(d) all the three coordinates change

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Q28. A cyclist, weighing a total of $80\,kg$ with the bicycle, pedals at a speed of $10\,m/s$. She stops pedalling at an instant which is taken to be $t = 0$. Due to the velocity dependent frictional force, her velocity is found to vary as $v(t) = \dfrac{10}{\left(1 + \dfrac{t}{30}\right)}\,m.s$, where t is measured in seconds. When the velocity drops to $8\,m/s$, she starts pedalling again to maintain a constant speed. The energy expended by her in 1 minute at this (new) speed, is

(a) $4\,kJ$ (b) $8\,kJ$ (c) $16\,kJ$ (d) $32\,kJ$

Q29. The number of linearly independent power series solutions, around $x = 0$, of the second order linear differential equation $x\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} + xy = 0$, is

(a) 0 (this equation does not have a power series solution)

(b) 1

(c) 2

(d) 3

Q30. Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator

$$\left[\frac{1}{2m}p^2 + \beta x^2, \frac{1}{m}p^2 + \gamma x^2\right]$$

where β and γ are constants, is zero if

(a) $\gamma = \beta$ (b) $\gamma = 2\beta$ (c) $\gamma = \sqrt{2}\beta$ (d) $2\gamma = \beta$

Q31. Two point charges $+3Q$ and $-Q$ are placed at $(0,0,d)$ and $(0,0,2d)$ respectively, above an infinite grounded conducting sheet kept in the xy- plane. At a point $(0,0,z)$, where $z \gg d$, the electrostatic potential of this charge configuration would approximately be

(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{d^2}{z^3}Q$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2d}{z^2}Q$ (c) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{3d}{z^2}Q$ (d) $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{d^2}{z^3}Q$

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Q32. A rectangular piece of dielectric material is inserted partially into the (air) gap between the plates of a parallel plate capacitor. The dielectric piece will

(a) remain stationary where it is placed

(b) be pushed out from the gap between the plates

(c) be drawn inside the gap between the plates and its velocity does not change sign

(d) execute an oscillatory motion in the region between the plates

Q33. An electromagnetic wave is travelling in free space (of permittivity ε_0) with electric field

$$\vec{E} = \hat{k}E_0 \cos q(x - ct)$$

The average power (per unit area) crossing planes parallel to $4x + 3y = 0$ will be

(a) $\frac{4}{5}\varepsilon_0 c E_0^2$
(b) $\varepsilon_0 c E_0^2$
(c) $\frac{1}{2}\varepsilon_0 c E_0^2$
(d) $\frac{16}{25}\varepsilon_0 c E_0^2$

Q34. A plane electromagnetic wave from within a dielectric medium (with $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$) is incident on its boundary with air, at $z = 0$. The magnetic field in the medium is $\vec{H} = \hat{j}H_0 \cos\left(\omega t - kx - k\sqrt{3}\,z\right)$, where ω and k are positive constants.

The angles of reflection and refraction are, respectively,

(a) 45^0 and 60^0
(b) 30^0 and 90^0
(c) 30^0 and 60^0
(d) 60^0 and 90^0

Q35. The normalized wavefunction of a particle in three dimensions is given by $\psi(r,\theta,\varphi) = \frac{1}{\sqrt{8\pi a^3}}e^{-r/2a}$ where $a > 0$ is a constant. The ratio of the most probable distance from the origin to the mean distance from the origin, is

[You may use $\int\limits_0^\infty dx\, x^n e^{-x} = n!$]

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $\frac{3}{2}$
(d) $\frac{2}{3}$

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Q36. The state vector of a one-dimensional simple harmonic oscillator of angular frequency ω, at time $t = 0$, is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |2\rangle\right]$, where $|0\rangle$ and $|2\rangle$ are the normalized ground state and the second excited state, respectively. The minimum time t after which the state vector $|\psi(t)\rangle$ is orthogonal to $|\psi(0)\rangle$, is

(a) $\dfrac{\pi}{2\omega}$ (b) $\dfrac{2\pi}{\omega}$ (c) $\dfrac{\pi}{\omega}$ (d) $\dfrac{4\pi}{\omega}$
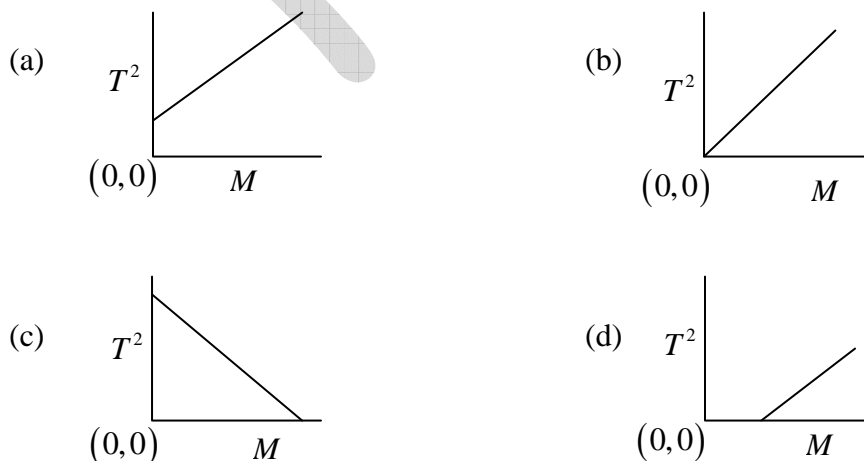
Q37. The normalized wavefunction in the momentum space of a particle in one dimension is $\phi(p) = \dfrac{\alpha}{p^2 + \beta^2}$, where α and β are real constants. The uncertainty Δx in measuring its position is

(a) $\sqrt{\pi}\,\dfrac{\hbar\alpha}{\beta^2}$ (b) $\sqrt{\pi}\,\dfrac{\hbar\alpha}{\beta^3}$ (c) $\dfrac{\hbar}{\sqrt{2}\beta}$ (d) $\sqrt{\dfrac{\pi}{\beta}}\,\dfrac{\hbar\alpha}{\beta}$

Q38. The full scale voltage of an n-bit Digital-to-Analog Convener is V. The resolution that can be achieved in it is

(a) $\dfrac{V}{(2^n - 1)}$ (b) $\dfrac{V}{(2^n + 1)}$ (c) $\dfrac{V}{2^{2n}}$ (d) $\dfrac{V}{n}$

Q39. The spring constant k of a spring of mass m_s is determined experimentally by loading the spring with mass M and recording the time period T, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is

(a)

(b)

(c)

(d)

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Q40. A Zener diode with an operating voltage of $10\,V$ at $25^0 C$ has a positive temperature coefficient of 0.07% per $^0 C$ of the operating voltage. The operating voltage of this Zener diode at $125^0 C$ is

(a) $12.0\,V$ (b) $11.7\,V$ (c) $10.7\,V$ (d) $9.3\,V$

Q41. The dispersion relation of a gas of spin $\dfrac{1}{2}$ fermions in two dimensions is $E = \hbar v \left|\vec{k}\right|$, where E is the energy, $\vec{k}$ is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is $\in_F$, the number of particles per unit area is

(a) $\dfrac{\in_F}{\left(4\pi v \hbar\right)}$ (b) $\dfrac{\in_F^3}{\left(6\pi^2 v^3 \hbar^3\right)}$ (c) $\dfrac{\pi \in_F^{3/2}}{\left(3 v^3 \hbar^3\right)}$ (d) $\dfrac{\in_F^2}{\left(2\pi v^2 \hbar^2\right)}$

Q42. The relation between the internal energy U, entropy S, temperature T, pressure p, volume V, chemical potential μ and number of particles N of a thermodynamic system is $dU = TdS - pdV + \mu dN$. That U is an exact differential implies that

(a) $-\left.\dfrac{\partial p}{\partial S}\right|_{V,N} = \left.\dfrac{\partial T}{\partial V}\right|_{S,N}$ (b) $p\left.\dfrac{\partial U}{\partial T}\right|_{S,N} = S\left.\dfrac{\partial U}{\partial V}\right|_{S,\mu}$

(c) $p\left.\dfrac{\partial U}{\partial T}\right|_{S,N} = -\dfrac{1}{T}\left.\dfrac{\partial U}{\partial V}\right|_{S,\mu}$ (d) $\left.\dfrac{\partial p}{\partial S}\right|_{V,N} = \left.\dfrac{\partial T}{\partial V}\right|_{S,N}$

Q43. The number of microstates of a gas of N particles in a volume V and of internal energy U, is given by

$$\Omega(U,V,N) = (V - Nb)^N \left(\frac{aU}{N}\right)^{3N/2}$$

(where a and b are positive constants). Its pressure P, volume V and temperature T, are related by

(a) $\left(P + \dfrac{aN}{V}\right)(V - Nb) = Nk_B T$ (b) $\left(P - \dfrac{aN}{V^2}\right)(V - Nb) = Nk_B T$

(c) $PV = Nk_B T$ (d) $P(V - Nb) = Nk_B T$

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Q44. Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1 respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\rightarrow 1}$ and $W_{1\rightarrow 0}$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$. Which of the following equations hold in equilibrium?

(a) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0}$

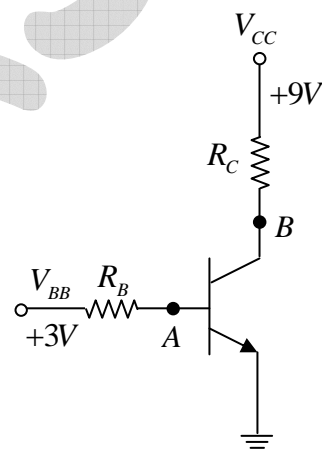(b) $P_0 n W_{0\rightarrow 1} = P_1 n W_{1\rightarrow 0}$

(c) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0} - P_1 n W_{1\rightarrow 0}$

(d) $P_0 n W_{0\rightarrow 1} = P_1 W_{1\rightarrow 0} + P_1 n W_{1\rightarrow 0}$

Q45. In the circuit below the voltages V_{BB} and V_{CC} are kept fixed, the voltage measured at B is a constant, but that measured at A fluctuates between a few μV to a few mV.

From these measurements it may be inferred that the

(a) base is open internally

(b) emitter is open internally

(c) collector resistor is open

(d) base resistor is open

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PART 'C'

Q46. Consider a set of particles which interact by a pair potential $V = ar^6$ where r is the inter-particle separation and $a > 0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{2}{3}$

Q47. In an inertial frame S, the magnetic vector potential in a region of space is given by $\vec{A} = az\hat{i}$ (where a is a constant) and the scalar potential is zero. The electric and magnetic fields seen by an inertial observer moving with a velocity $v\hat{i}$ with respect to S, are, respectively [In the following $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$]

(a) 0 and $\gamma a\hat{j}$ (b) $-va\hat{k}$ and $\gamma a\hat{i}$

(c) $v\gamma a\hat{k}$ and $v\gamma a\hat{j}$ (d) $v\gamma a\hat{k}$ and $\gamma a\hat{j}$

Q48. In the rest frame S_1 of a point particle with electric charge q_1 another point particle with electric charge q_2 moves with a speed v parallel to the x-axis at a perpendicular distance l. The magnitude of the electromagnetic force felt by q_1 due to q_2 when the distance between them is minimum, is [In the following $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$]

(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{\gamma l^2}$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma q_1 q_2}{l^2}$

(c) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma q_1 q_2}{l^2}\left(1 + \dfrac{v^2}{c^2}\right)$ (d) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{\gamma l^2}\left(1 + \dfrac{v^2}{c^2}\right)$

Q49. A circular current carrying loop of radius a carries a steady current. A constant electric charge is kept at the centre of the loop. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$ respectively, at a distance d vertically above the centre of the loop satisfy

(a) $\vec{E} \perp \vec{B}$ (b) $\vec{E} = 0$ (c) $\vec{\nabla}\left(\vec{E} \cdot \vec{B}\right) = 0$ (d) $\vec{\nabla} \cdot \left(\vec{E} \times \vec{B}\right) = 0$

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Q50. A phase shift of 30^0 is observed when a beam of particles of energy $0.1 MeV$ is scattered by a target. When the beam energy is changed, the observed phase shift is 60^0. Assuming that only s-wave scattering is relevant and that the cross-section does not change with energy, the beam energy is

(a) $0.4\ MeV$ (b) $0.3\ MeV$ (c) $0.2\ MeV$ (d) $0.15\ MeV$

Q51. Consider an element $U(\varphi)$ of the group $SU(2)$, where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as $U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(gj)$. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be

(a) positive definite   (b) real symmetric   (c) hermitian   (d) anti-hermitian

Q52. The differential equation $\dfrac{dy(x)}{dx} = \alpha x^2$, with the initial condition $y(0) = 0$, is solved using Euler's method. If $y_E(x)$ is the exact solution and $y_N(x)$ the numerical solution obtained using n steps of equal length, then the relative error $\left| \dfrac{(y_N(x) - y_E(x))}{y_E(x)} \right|$ is proportional to

(a) $\dfrac{1}{n^2}$ (b) $\dfrac{1}{n^3}$ (c) $\dfrac{1}{n^4}$ (d) $\dfrac{1}{n}$

Q53. The interval $[0,1]$ is divided into n parts of equal length to calculate the integral $\int_0^1 e^{i2\pi x} dx$ using the trapezoidal rule. The minimum value of n for which the result is exact, is

(a) 2   (b) 3   (c) 4   (d) ∞

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Q54. The generating function $G(t, x)$ for the Legendre polynomials $P_n(t)$ is

$$G(t, x) = \frac{1}{\sqrt{1 - 2xt + x^2}} = \sum_{n=0}^{\infty} x^n P_n(t), \quad \text{for } |x| < 1$$

If the function $f(x)$ is defined by the integral equation $\int_0^x f(x')\, dx' = xG(1, x)$, it can be
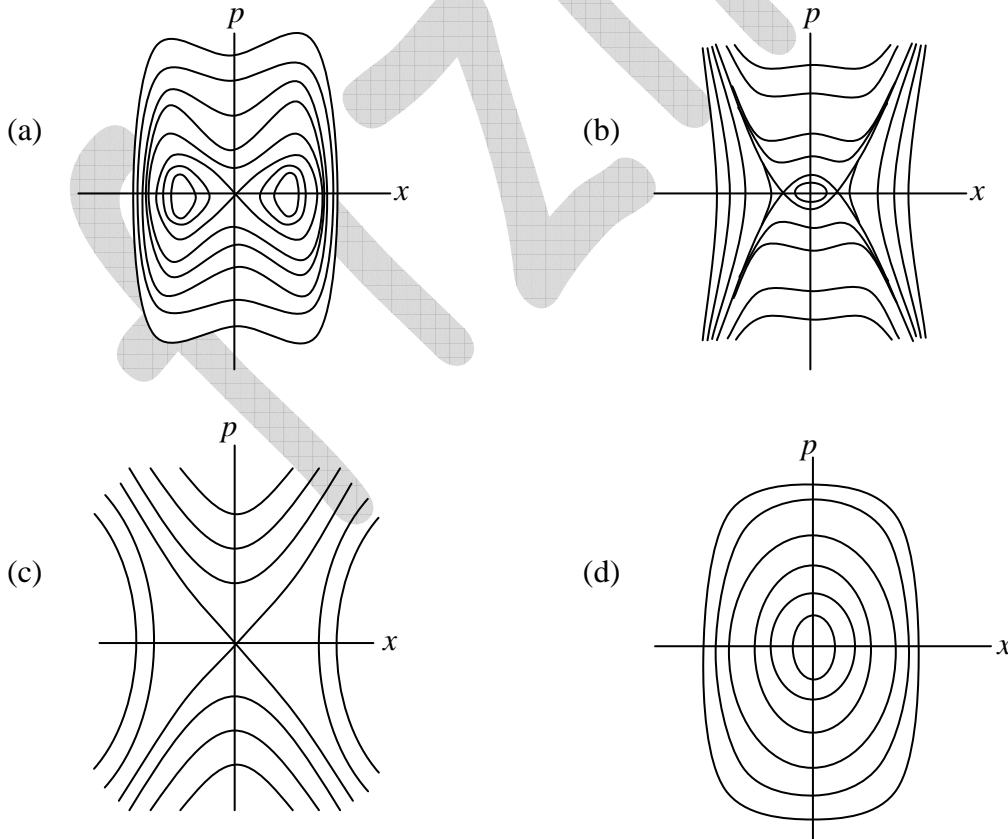
expressed as

(a) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m\left(\frac{1}{2}\right)$

(b) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m(1)$

(c) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(1) P_m(1)$

(d) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(0) P_m(1)$

Q55. A particle moves in one dimension in a potential $V(x) = -k^2 x^4 + \omega^2 x^2$ where k and ω are constants. Which of the following curves best describes the trajectories of this system in phase space?

(a)

(b)

(c)

(d)

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Q56. Let (x, p) be the generalized coordinate and momentum of a Hamiltonian system. If new variables (X, P) are defined by $X = x^\alpha \sinh(\beta p)$ and $P = x^\gamma \cosh(\beta p)$, where α, β and γ are constants, then the conditions for it to be a canonical transformation, are

(a) $\alpha = \dfrac{1}{2\beta}(\beta + 1)$ and $\gamma = \dfrac{1}{2\beta}(\beta - 1)$

(b) $\beta = \dfrac{1}{2\gamma}(\alpha + 1)$ and $\gamma = \dfrac{1}{2\alpha}(\alpha - 1)$

(c) $\alpha = \dfrac{1}{2\beta}(\beta - 1)$ and $\gamma = \dfrac{1}{2\beta}(\beta + 1)$

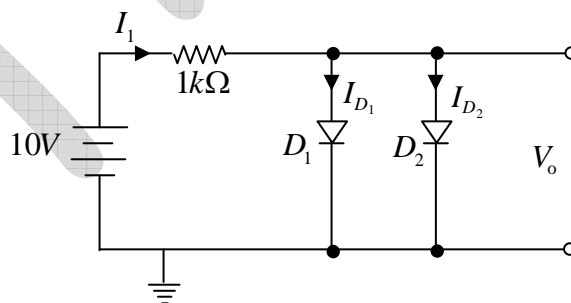(d) $\beta = \dfrac{1}{2\gamma}(\alpha - 1)$ and $\gamma = \dfrac{1}{2\alpha}(\alpha + 1)$

Q57. A closed system having three non-degenerate energy levels with energies $E = 0, \pm \in$, is at temperature T. For $\in = 2k_B T$, the probability of finding the system in the state with energy $E = 0$, is

(a) $\dfrac{1}{(1 + 2\cosh 2)}$
(b) $\dfrac{1}{(2\cosh 2)}$
(c) $\dfrac{1}{2}\cosh 2$
(d) $\dfrac{1}{\cosh 2}$

Q58. Two non-degenerate energy levels with energies 0 and $\in$ are occupied by N non-interacting particles at a temperature T. Using classical statistics, the average internal energy of the system is

(a) $\dfrac{N \in}{\left(1 + e^{\in / k_B T}\right)}$
(b) $\dfrac{N \in}{\left(1 - e^{\in / k_B T}\right)}$
(c) $N \in e^{-\in / k_B T}$
(d) $\dfrac{3}{2}Nk_B T$

Q59. In the circuit below, D_1 and D_2 are two silicon diodes with the same characteristics. If the forward voltage drop of a silicon diode is $0.7\ V$ then the value of the current $I_1 + I_{D_1}$ is

(a) $18.6\ mA$
(b) $9.3\ mA$
(c) $13.95\ mA$
(d) $14.65\ mA$

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Q60. The Hamiltonian of a two-level quantum system is $H = \frac{1}{2}\hbar\omega\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ possible initial state in which the probability of the system being in that quantum state does not change with time, is

(a) $\begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix}$
(b) $\begin{pmatrix} \cos\frac{\pi}{8} \\ \sin\frac{\pi}{8} \end{pmatrix}$
(c) $\begin{pmatrix} \cos\frac{\pi}{2} \\ \sin\frac{\pi}{2} \end{pmatrix}$
(d) $\begin{pmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{pmatrix}$

Q61. Consider a one-dimensional infinite square well

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a, \\ \infty & \text{otherwise} \end{cases}$$

If a perturbation

$$\Delta V(x) = \begin{cases} V_0 & \text{for } 0 < x < a/3, \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state, to first order in ΔV, is nearest to

(a) V_0
(b) $0.16\, V_0$
(c) $0.2\, V_0$
(d) $0.33\, V_0$

Q62. The energy eigenvalues E_n of a quantum system in the potential $V = cx^6$ (where $c > 0$ is a constant), for large values of the quantum number n, varies as

(a) $n^{4/3}$
(b) $n^{3/2}$
(c) $n^{5/4}$
(d) $n^{6/5}$

Q63. Consider a quantum system of non-interacting bosons in contact with a particle bath. The probability of finding no particle in a given single particle quantum state is 10^{-6}. The average number of particles in that state is of the order of

(a) 10^3
(b) 10^6
(c) 10^9
(d) 10^{12}

Q64. The sensitivity of a hot cathode pressure gauge is $10\,\text{mbar}^{-1}$. If the ratio between the numbers of the impinging charged particles to emitted electrons is $1:10$, then the pressure

(a) $10\,\text{mbar}$
(b) $10^{-1}\,\text{mbar}$
(c) $10^{-2}\,\text{mbar}$
(d) $10^2\,\text{mbar}$

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Q65. Two physical quantities T and M are related by the equation $T = \dfrac{2\pi}{a}\sqrt{\dfrac{M+b}{2}}$, where a and b are constant parameters. The variation of T as a function of M was recorded in an experiment to determine the value of a graphically. Let m be the slope of the straight line when T^2 is plotted $vs\, M$, and δm be the uncertainty in determining it. The uncertainty in determining a is

(a) $\dfrac{a}{2}\left(\dfrac{\delta m}{m}\right)$ 　　(b) $a\left(\dfrac{\delta m}{m}\right)$ 　　(c) $\dfrac{b}{2a}\left(\dfrac{\delta m}{m}\right)$ 　　(d) $\dfrac{2\pi}{a}\left(\dfrac{\delta m}{m}\right)$

Q66. The circuit below comprises of D-flip flops. The output is taken from Q_3, Q_2, Q_1 and Q_0 as shown in the figure.
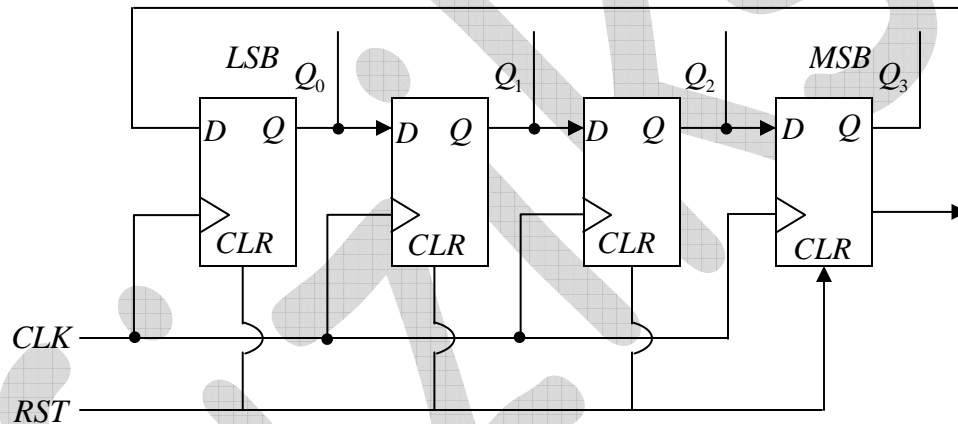
the binary number given by the string $Q_3, Q_2, Q_1 Q_0$ changes for every clock pulse that is applied to the CLK input. If the output is initialized at 0000, the the corresponding sequence of decimal numbers that repeats itself, is

(a) $3, 2, 1, 0$ 　　　　　　　　　(b) $1, 3, 7, 14, 12, 8$

(c) $1, 3, 7, 15, 12, 14, 0$ 　　　　(d) $1, 3, 7, 15, 14, 12, 8, 0$

Q67. The spin-parity assignments for the ground and first excited states of the isotope $^{57}_{28}Ni$, in the single particle shell model, are

(a) $\left(\dfrac{1}{2}\right)^-$ and $\left(\dfrac{3}{2}\right)^-$ 　　　　　　(b) $\left(\dfrac{5}{2}\right)^+$ and $\left(\dfrac{7}{2}\right)^+$

(c) $\left(\dfrac{3}{2}\right)^+$ and $\left(\dfrac{5}{2}\right)^+$ 　　　　　　(d) $\left(\dfrac{3}{2}\right)^-$ and $\left(\dfrac{5}{2}\right)^-$

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Q68. The first excited state of the rotational spectrum of the nucleus $^{238}_{92}U$ has an energy $45\,keV$ above the ground state. The energy of the second excited state (in keV) is

(a) 150   (b) 120   (c) 90   (d) 60

Q69. Which of the following process is not allowed by the strong interaction but is allowed by the weak interaction?

(a) $K^0 + \pi^0 \rightarrow \bar{K}^0 + \pi^+ + \pi^-$ (b) $p + n \rightarrow d + p + \bar{p}$

(c) $\Delta^+ + K^0 \rightarrow p + n$ (d) $p + \Delta^+ \rightarrow \bar{n} + \Delta^{++}$

Q70. A crystal of MnO has $NaCl$ structure. It has a paramagnetic to anti- ferromagnetic transition at $120\,K$. Below $120\,K$, the spins within a single $[111]$ planes are parallel but the spins in adjacent $[111]$ planes are antiparallel. If neutron scattering is used to determine the lattice constants, respectively, d and d', below and above the transition temperature of MnO then

(a) $d = \dfrac{d'}{2}$ (b) $d = \dfrac{d'}{\sqrt{2}}$ (c) $d = 2d'$ (d) $d = \sqrt{2}d'$

Q71. A metallic nanowire of length l is approximated as a one-dimensional lattice of N atoms with lattice spacing a. If the dispersion of electrons in the lattice is given as $E(k) = E_0 - 2t\cos ka$, where E_0 and t are constants, then the dnsity of states inside the nanowire depends on E as

(a) $N^3 \sqrt{\dfrac{t^2}{E - E_0}}$ (b) $\sqrt{\left(\dfrac{E - E_0}{2t}\right)^2 - 1}$

(c) $N^3 \sqrt{\dfrac{E - E_0}{t^2}}$ (d) $\dfrac{N}{\sqrt{(2t)^2 - (E - E_0)^2}}$

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Q72. Consider a two-dimensional material of length l and width w subjected to a constant magnetic field B applied perpendicular to it. The number of change carries per unit area may be expressed as $n = k|q|\dfrac{B}{(2\pi\hbar)}$, where k is appositive real number and q is the carrier charge. Then the Hall resistivity ρ_{xy} is

(a) $\dfrac{2\pi\hbar k}{q^2}\sqrt{\dfrac{l}{w}}$ 　　 (b) $\dfrac{2\pi\hbar}{kq^2}\sqrt{\dfrac{w}{l}}$ 　　 (c) $\dfrac{2\pi\hbar}{kq^2}$ 　　 (d) $\dfrac{2\pi\hbar k}{q^2}$

Q73. The Zeeman shift of the energy of a state with quantum numbers L, S, J and m_J is

$$H_z = \frac{m_J \mu_B B}{J(J+1)}\left(\left\langle \vec{L}.\vec{J}\right\rangle + g_s \left\langle \vec{S}.\vec{J}\right\rangle\right)$$

Where B is the applied magnetic field, g_s is the g-factor for the spin and $\dfrac{\mu_B}{h} = 1.4\ MHz - G^{-1}$, where h is the Planck constant. The approximate frequency shift of the $S = 0, L = 1$ and $m_J = 1$ state, at a magnetic field of $1G$, is

(a) $10\ MHz$ 　　 (b) $1.4\ MHz$ 　　 (c) $5\ MHz$ 　　 (d) $2.8\ MHz$

Q74. The separations between the adjacent levels of a normal multiplet are found to be $22\ cm^{-1}$ and $33\ cm^{-1}$. Assume that the multiplet is described well by the $L-S$ coupling scheme and the Lande's interval rule, namely $E(J) - E(J-1) = AJ$, where A is a constant. The term notaions for this multiplet is

(a) $^3P_{0,1,2}$ 　　 (b) $^3F_{2,3,4}$ 　　 (c) $^3G_{3,4,5}$ 　　 (d) $^3D_{1,2,3}$

Q75. If the fine structure splitting between the $2\ ^2P_{3/2}$ and $2\ ^2P_{3/2}$ levels in the hydrogen atom is $0.4\ cm^{-1}$, the corresponding splitting in Li^{2+} will approximately be

(a) $1.2\ cm^{-1}$ 　　 (b) $10.8\ cm^{-1}$ 　　 (c) $32.4\ cm^{-1}$ 　　 (d) $36.8\ cm^{-1}$

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